Digital signal processing (DSP) with discrete multitone (DMT), digital subscriber line (DSL) and the orthogonal frequency division multiplexing (OFDM) systems

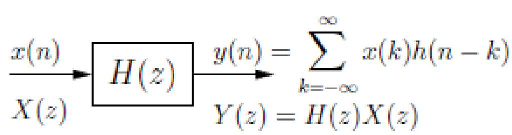
Kuldeep bhardwaj
Ph.d (ECE) scholar
Sai nath university, Ranchi (Jharkhand)

Abstract - The theory of multirate digital signal processing (DSP) has traditionally been applied to the contexts of filter banks and wavelets. These play a very important role in signal decomposition, analysis, modeling and reconstruction. Many areas of signal processing would be hard to envision without the use of digital filter banks. This is especially true for audio, video and image compression, digital audio processing, signal denoising, adaptive and statistical signal processing. However, multirate DSP has recently found increasing application in digital communications as well. Multirate building blocks are the crucial ingredient in many modern communication systems, for example, the discrete multitone (DMT), digital subscriber line (DSL) and the orthogonal frequency division multiplexing(OFDM) systems as well as general filter bank precoders, just to name a few.

Keywords: DSP, Digital Audio processing, filters and scalar signals

I. I. INTRODUCTION

The signals of interest in digital signal processing are discrete sequences of real or complex numbers denoted by $x(n),\ y(n),$, etc. The sequence $x(n)$ is often obtained by sampling a continuous-time signal $x_c(t)$. The majority of natural signals (like the audio signal reaching our ears or the optical signal reaching our eyes) are continuous-time. However, in order to facilitate their processing using DSP techniques, they need to be sampled and converted to digital signals. This conversion also includes signal quantization, i.e.,discretization in amplitude, however in practice it is safe to assume that the amplitude of $x(n)$ can be any real or complex [1]



number. Signal processing analysis is often simplified by considering the frequency domain representation of signals and systems. Commonly used alternative representations of x(n) are its z-transform $X(z)$ and the discrete-time Fourier transform $X(e^{j\omega})$. The z-transform is defined as $X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$, and $X(e^{j\omega})$ is nothing but $X(z)$ evaluated on the unit circle $z = e^{j\omega}$.

Multirate DSP systems are usually composed of three basic building blocks, operating on a discrete-time signal $x(n)$. Those are the linear time invariant (LTI) filter, the decimator and the expander. An LTI filter, like the one shown in Fig.1.1, is characterized by its impulse response $h(n)$, or equivalently by its z-transform (also called the transfer function) $H(z)$. Examples of the M-fold decimator and expander for M = 2 are shown in Fig.1.2. The rate of the signal at the output of an expander is M times higher than the rate at its input, while the converse is true for decimators. That is why the systems containing expanders and decimators are called 'multirate' systems. Fig.1.2 demonstrates the behavior of the decimator and the expander in both the time and the frequency domains. In the z-domain this is described by

$$X_E(z) = [X(z)]_{\uparrow M} = X(z^M) \qquad \text{for } M\text{-fold expander, and}$$

$$X_D(z) = [X(z)]_{\downarrow M} = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{\frac{1}{M}} e^{-\frac{j2\pi k}{M}}) \qquad \text{for } M\text{-fold decimator.}$$

The systems shown in Figs.1.1 and 1.2 operate on scalar signals and thus are called single input— single output (SISO) systems. The extensions to the case of vector signals are rather straightforward: the decimation and the expansion are performed on each element separately. The corresponding vector sequence decimators/expanders are denoted within square boxes in block diagrams. In Fig.1.3 this is demonstrated for vector expanders. The LTI systems operating on vector signals are called multiple input—multiple output (MIMO) systems and they are characterized by a (possibly rectangular) matrix transfer function H(z).

II. II. LITERATURE REVIEW

The vector signals are sometimes obtained from the corresponding scalar signals by blocking. Conversely, the scalar signals can be recovered from the vector signals by unblocking. The blocking/unblocking operations can be defined using the delay or the advance chains [2], thus leading to two similar definitions. One way of defining these operations is shown in Fig.1.4, while the other is obtained trivially by switching the delay and the advance operators. Instead of drawing the complete delay/advance chain structure, we often use the simplified block notation as in Fig.1.4. It is usually clear from the context which of the two definitions

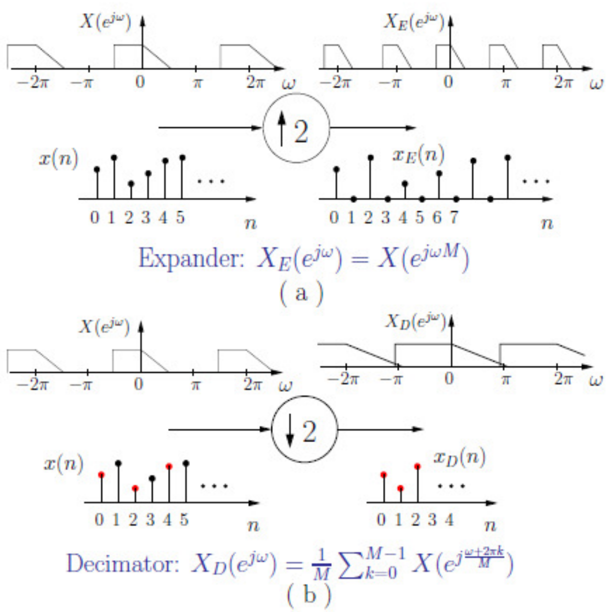


Figure 1.2: Multirate building blocks: (a) 2-fold expander and (b) decimator.

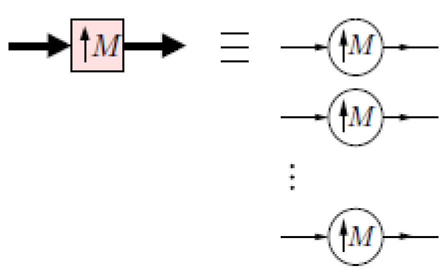


Figure 1.3: The definition and the notation of the vector signal expander.

of the unblocking and blocking operations is employed. A very useful tool in multirate signal processing is the so-called polyphase representation of signals and systems. It facilitates considerable simplifications of theoretical results as well as efficient implementation of multirate systems. Since polyphase representation will play an important role in the rest of the thesis, here we take a moment to formally define it. Consider an LTI system with a transfer function $H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}$ and suppose we are given an integer M. We can decompose $H(z)$ as

$$H(z) = \sum_{m=0}^{M-1} z^{-m} \sum_{n=-\infty}^{\infty} h(nM+m)z^{-nM} = \sum_{m=0}^{M-1} z^{-m} H_m(z^M) \quad \text{(Type 1 decomposition).}$$

Note that this is equivalent to dividing the impulse response h(n) into M nonoverlaping groups of samples

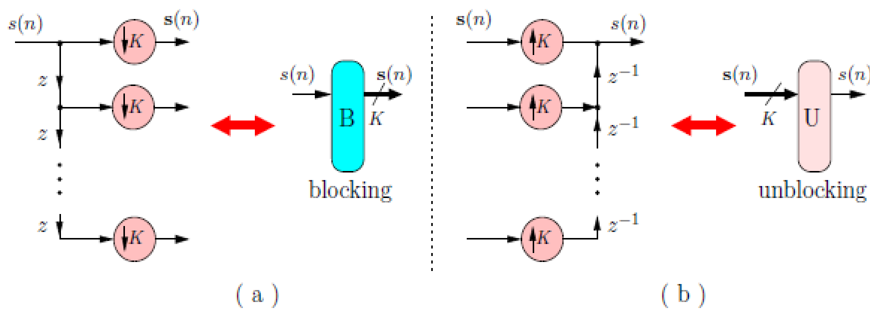


Figure 1.4: Notations: blocking (a) and unblocking (b) operations.

$h_m(n)$, obtained from $h(n)$ by M-fold decimation starting from sample m. In other words, $h(n)$ can be obtained by combining sequences $h_m(n)$ through the unblocking structure shown in Fig.1.4(b). Subsequences $h_m(n)$ and the corresponding z-transforms defined in (1.3) are called the Type 1 polyphase components of $H(z)$ with respect to M. A variation of (1.3) is obtained if we decimate h(n) starting from sample $-m$, for $0 \leq m \leq M-1$. This gives rise to Type 2 polyphase components $\bar{H}_m(z)$: [3]

$$H(z) = \sum_{m=0}^{M-1} z^{m} \bar{H}_m(z^M) \quad \text{(Type 2 decomposition).}$$

The polyphase notation will be used again very soon in Section 1.3.2 when we discuss the use of filter bank precoders in modern digital communications. However, it is also an important tool in the rest of the thesis The reader will therefore often be referred to the results from this section. In the following we first describe the notion that plays the central role in Chapters 2 and 3, namely, the concept of biorthogonal partners.

III. METHODOLOGY

Polyphase matrix notation. The overdecimated filter banks in Fig.1.10 are equivalent to the respective systems shown on the right-hand side. Rectangular matrices $\mathbf{R}(z)$ and $\mathbf{E}(z)$ are called polyphase matrices for the corresponding banks of filters. Using the blocking and unblocking definitions as in Fig.1.4 it can be shown that the kth column of $\mathbf{R}(z)$ consists of Type 1 polyphase components of $F_k(z)$ and that the $j\text{th}$ row of $\mathbf{E}(z)$ consists of Type 2 polyphase components of $H_j(z)$, both with respect to P. In other words, let

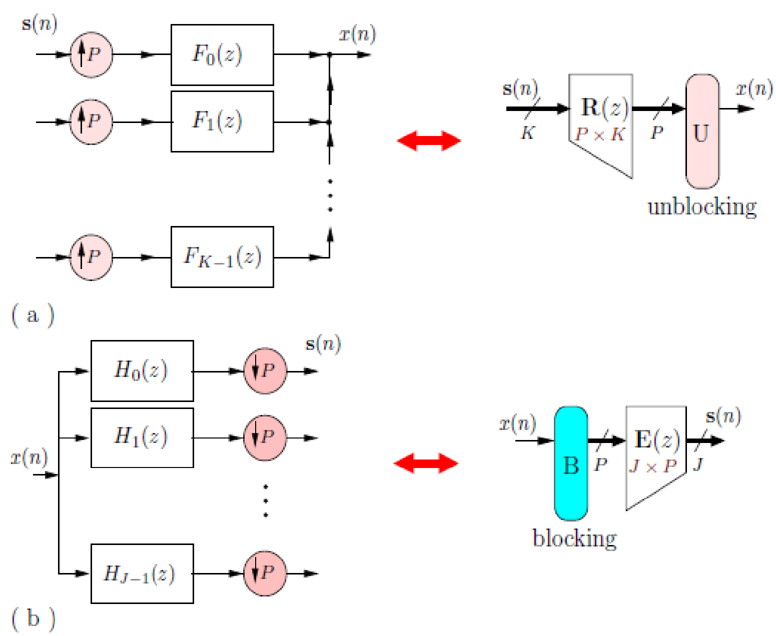


Figure 1.10: Polyphase representations of (a) filter bank precoder and (b) analysis filter bank.

$\mathbf{d}(z) = [1 \quad z^{-1} \quad \cdots \quad z^{-(P-1)}]$; then we have

$$[F_0(z) \quad F_1(z) \quad \cdots \quad F_{K-1}(z)] = \mathbf{d}(z)\mathbf{R}(z^M), \quad \text{and} \quad [H_0(z) \quad H_1(z) \quad \cdots \quad H_{J-1}(z)]^T = \mathbf{E}(z^M)\tilde{\mathbf{d}}(z). \qquad (1.6)$$

In addition to providing a compact notation, the structures on the right-hand side of Fig.1.10 are also efficient from the computational point of view (they promote parallel computations). For example, the rate of the vector signal at the entrance of $\mathbf{R}(z)$ is P times lower than that of the signal at the entrance of the filters $\{F_k(z)\}$. Finally, note that even though the expanders and decimators do not appear explicitly in the diagrams on the right-hand side of Fig.1.10, these are indeed multirate systems by the virtue of the fact that the combined rates at the input and at the output of the systems are not equal.

Significance of filter bank precoders. Precoders find use in solving some of the following problems.

1. Blind channel equalization. If the channel is unknown but is assumed to be of finite length, its interference effect is modeled as an unknown FIR filter that should be undone at the receiver. Redundancy introduced by the precoder at the transmitter makes it easier for the receiver to 'guess' this linear transform [4,5].

2. Equalization of ill-behaved channels. If the channel has zeros outside or close to the unit circle, the inverse filtering required for equalization might be noncausal, unstable, or simply very sensitive to the input noise. Introducing certain redundancy at the transmitter helps avoid inverse filtering altogether.

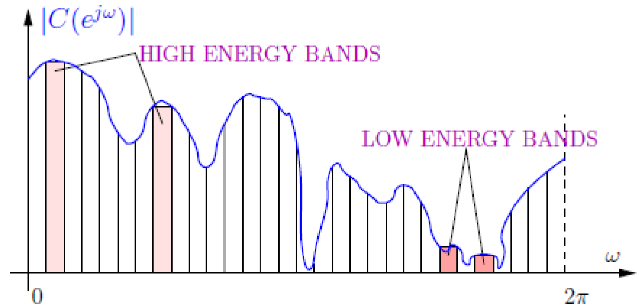


Figure 1.11: Channel magnitude response divided in frequency bands.

Consequently, these alternative equalizers usually perform better in the presence of noise.

3. Power and bit allocation in frequency bands. Some filter bank precoders together with the corresponding equalization structures at the receiver effectively divide the channel frequency response into a certain number of nonoverlaping channels, corresponding to different frequency bands (see Fig.1.11). The data is then divided according to certain criteria and sent over these independent channels. In order to achieve better performance of the overall system, it proves beneficial to allocate bits and power nonuniformly across different bands. The optimal allocation algorithm is a function of the corresponding channel energies and the noise power spectral density (PSD).

4. User separation in multiuser systems. Consider the 'uplink scenario' when M different users simultaneously send messages to the common receiver. How to extract the message from user m, i.e.,cancel the interference from the other users without compromising the quality of the desired signal.. It is further complicated when different users communicate through different, possibly unknown and time-varying channels. One approach involves using a filter bank precoder for introducing the controlled redundancy which serves as a signature distinguishing the desired user from the interfering ones [4,5].

IV. Findings

As mentioned previously, they arise in many different contexts. The central application considered here is that of MIMO channel equalization, especially with fractionally spaced equalizers. Returning to the general communication system from Fig.1.9, this scenario corresponds to communicating with vector signals and sampling the received signal at rate q/T, for some integer $q > 1$. In this context we assume that no additional redundancy has been inserted in the data stream and that there is no precoding in the system.

V. Conclusion

Further, initiated by deriving the comprehensive theory of MIMO biorthogonal partners and answering some of the most important questions inherited from the scalar case. What are the conditions for the existence of MIMO biorthogonal partners and what is their most general form? Under what conditions do rational matrix transfer functions have polynomial (or FIR) biorthogonal partners? When are these FIR partners unique? How to construct the most general FIR partner (of a given order)? After deriving the theoretical framework of MIMO biorthogonal partners, we consider some of their applications. In MIMO channel equalization, we exploit the inherent non-uniqueness of biorthogonal partners and construct fractionally spaced equalizers (FSEs) that perfectly eliminate the inter-symbol interference (ISI) introduced by the channel, and at the same time minimize the noise power at the receiver. Comparing the performance of these flexible FSEs to the symbol-spaced solutions and FSEs without noise optimization, we conclude that significant improvements in performance are possible with minimal or no increase in the receiver complexity. Several other applications of MIMO biorthogonal partners are considered next. We review their role in the least squares approximation of vector signals. In this context the least squares problem is limited to that of finding the approximation for a vector signal $\mathbf{x}(n)$ within a class of signals described by a multirate model. The optimal solution involves a certain form of biorthogonal partners. Finally, we consider the relation between biorthogonal partners and multiwavelets, especially the multi wavelet prefiltering.

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