Analysis and Simulation of LCR, AFD and Capacity of Nakagami Fading Channel in MATLAB Environment

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Abstract- The paper simulated and analyzed the statistical perfomances of the Nakagami fading channel in wireless communication with MATLAB, including the Level Crossing Rates and Average Fade Durations on the Maximal-Ratio Combining diversity. The objective of this paper is also to analysis channel capacity for correlated MIMO Nakagami – m fading with Waterfilling Algorithm. Multiple Input and Multiple Output (MIMO) system have the potential to achieve very high capacities, depending on the propagation environment. A MIMO system provides multiple independent transmission channels, thus, leading (under certain conditions) to a channel capacity that increases linearly with the number of antenna elements. It is well known that the employment of multiple antennas at both the transmitter and receiver increases the overall system capacity.

Keywords - LCR, AFD, MIMO, Nakagami Fading, MATLAB

I. INTRODUCTION

The first work to researching and developing of digital mobile communication engineering is understanding mobile channel characteristics itself. In the propagation environment of digital mobile communication, the signal received by the mobile may consist of a large number of multipath component due to reflection, diffraction and scattering between the transmitter and the receiver. The randomly distributed phase, amplitudes, and angles of arrival of multipath component combine at the receiver give a resultant signal strength which can change rapid over a small travel distance and be named fast fading [1]. There are various fading channels in digital communication but here we focused on Nakagami fading channel model. Nakagami fading could represent various fading condition in wireless channel, so simulation and analysis of the statistical characters of the Nakagami fading channel are very important. This paper made use of MATLAB to simulate and analyze the complex envelop of received signal, the Level Crossing Rates arid Average Fade Durations on the Maximal-Ratio Combining diversity and also show overview of some important theoretical concepts channel capacity of MIMO systems over Nakagami fading channel. In the never-ending search for increased capacity in a wireless communication channel it has been shown that by using MIMO (Multiple Input Multiple Output) system architecture it is possible to increase that capacity substantially. Usually fading is considered as a problem in wireless communication but MIMO channels uses the fading to increase the capacity. In terms of queuing theory, it is known that gamma distribution arises naturally in processes for which waiting times between poison distributed events are relevant [2].

II. CONCEPT OF NAKAGAMI FADING CHANNEL

With Nakagami-m distribution, sometimes denoted by m-distribution, a wide class of fading channel conditions can be modeled as explained in the introduction. This fading distribution has gained a lot of attention lately, since the Nakagami-m distribution often gives the best fit to land mobile and indoor mobile multipath propagation as well as scintillating ionospheres radio links [3]. More recent studies also showed that Nakagami-m gives the best fit for satelliteto-indoor and satellite-to-outdoor radio wave propagation [4]. The pdf (probability density function) of Nakagami channel is given as [5, 6]:

$$\rho_{\gamma}(\gamma) = \frac{m^{m}\gamma^{m-1}}{\bar{\gamma}^{m}\Gamma(m)} exp\left(\frac{-m\gamma}{\bar{\gamma}}\right), \gamma \geq 0 \qquad ..(1)$$

Where, $\bar{\gamma}$ is average fading power, $\gamma \geq 0$ as the channel amplitude, $exp(*)$ is the expectation operator, $\Gamma(*)$ is the gamma distribution and m is the nakagami fading parameter.

III. MIMO CHANNEL MODEL

Multiple inputs multiple outputs (MIMO) systems in wireless communications refer to any wireless communication system where at both sides of the communication path more than one antenna is used. The idea behind MIMO is that, the signals on the transmit (T_X) antennas at one end and the receive (R_X) antennas at the other end are "combined" in such a way that the quality (bit error rate or BER) or the data rate (bits/sec) of the communication for each MIMO user will be improved [7].

MIMO is based on main spatial multiplexing principle called Spatial Multiplexing. It is a transmission technique to transmit independent and separately encoded data signals, so-called streams, from each of the multiple transmit antennas. Therefore, the space dimension is reused, or multiplexed, more than one time [8].

In order to design efficient communication algorithms for MIMO systems and to understand the performance limits, it is important to understand the nature of the MIMO channel. For a system with M_T transmit antennas and M_R receive antennas, assuming frequency-flat fading over the bandwidth of interest, the MIMO channel at a given time Instant may be represented as an $M_R \times M_T$ matrix [6].

$$\boldsymbol{H} = \begin{bmatrix} H_{1,1} & \cdots & H_{1,M_T} \\ \vdots & \ddots & \vdots \\ H_{M_R,1} & \cdots & H_{M_R,M_T} \end{bmatrix} \qquad ..(2)$$

Where, M_R is the m^th^ term and M_T as n^th^, so $H_{m,n}$ is the (single-input single-output) channel gain between the m^{th} receive and nth transmit antenna pair. The nth column of H is often referred to as the spatial signature of the nth transmit antenna across the receive antenna array. The relative geometry of the M_T spatial signatures determines the distinguish ability of the signals launched from the transmit antennas at a receiver.

IV. WATERFILLING THEOREM AND CAPACITY OF MIMO CHANNEL

Waterfilling is a metaphor for the solution of several optimization problems related to channel capacity. The simplest physical example is perhaps the case of spectral allocation for maximal total capacity under a total power constraint [9].

The well – known classical waterfilling solution solves the problem of maximizing the mutual information between the input and the output of a channel composed of several sub channels (such as a frequency – selective sub channels arising from the use of multiple antennas at both sides of the link)[7][9] with a global power constraint at the transmitter. This capacity – achieving solution has the visual interpretation of pouring water over a surface given by the inverse of the sub channel gains hence the name waterfilling or waterpouring. The capacity of MIMO channel with waterfilling can be expressed as [7]

$$C = \sum_{1}^{m} \log_2\left[1 + \left(\frac{\rho_i}{\sigma^2}\right) * \rho_i{}^2\right] bps/Hz \qquad ..(3)$$

Above equation enables the visualization of the MIMO channel as a number of parallel SISO pipes with gain equal. Therefore to the respective Eigen values and its enables as to understand that the waterfilling capacity for MIMO channels is the sum of the capacities of the SISO equivalent parallel sub channels, obtained from performing SVD on MIMO channel matrix. If the channel is known at the transmitter, the capacity can be enhanced by using the good channels i.e. those with the highest gain by applying an unequal power distribution.

V. SIMULATION AND RESULTS

5.1 LCR and AFD for diversity in Nakagami fading channel

The level-crossing rates (LCRs) and average fade durations (AFDs) are two quantities which statistically characterize fading channel, these quantities reflect not only scattering environment but also velocity of mobile, and thus the second-order statistics of a fading channel. The LCR N_R (r) is defined as the number of times per second that the envelop of fading channel crosses a specified level *R* in a positive-going direction, the AFD τ_R(r) is defined as the average period of time for which the received signal is below a specified level R. Let *r* be the sampled value of the diversity combined envelope *R(t)* of a fading channel. The LCR $N_R(r)$ and ADF τ_R(r) are defined as a function of *r* by [10]

$$N_R(r) = \int_0^\infty \dot{r} p_{R,\dot{R}}(r,\dot{r}) d\dot{r} \qquad ..(4)$$

$$\tau_R(r) = F_R(r)/N_R(r) \qquad ..(5)$$

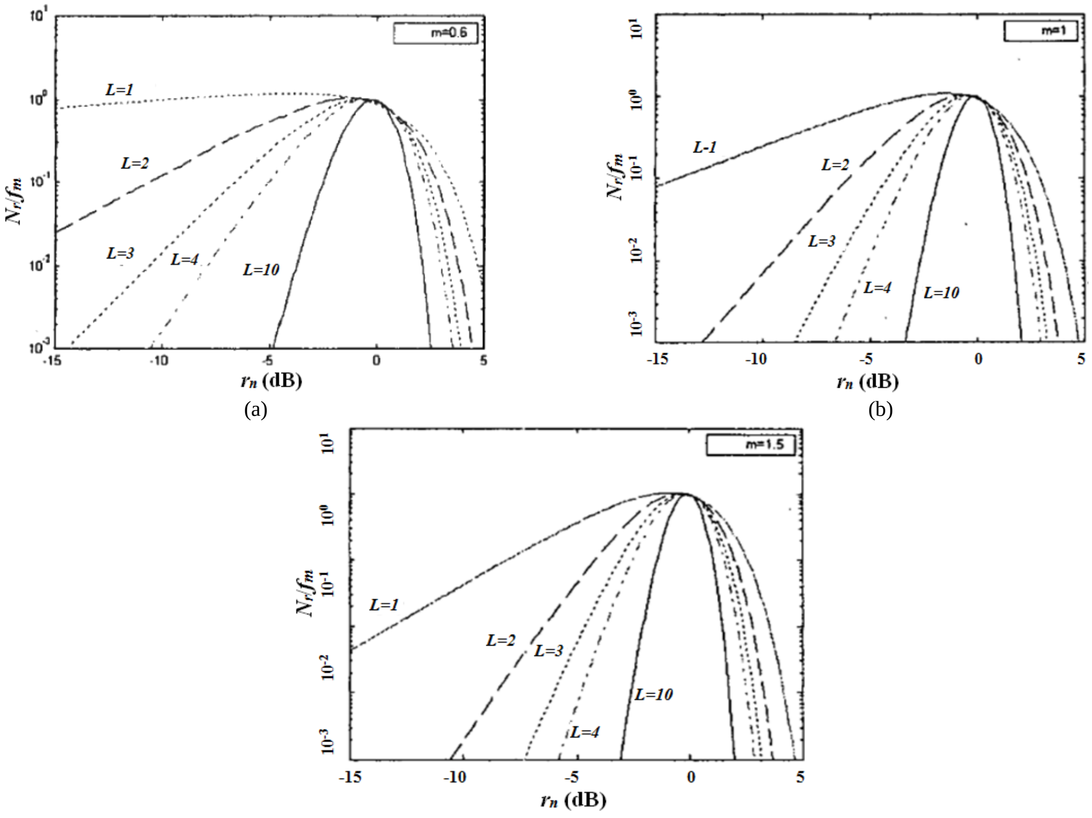
$F_R(r) = \int_0^r p_r(\alpha) d\alpha$ is the CDF of the fading channel, and $p_r(r)$ is the corresponding PDF. Rewriting of LCR in terms of $p_r(r)$ & conditional distribution $p_{\dot{R}}(\dot{r}|r)$ and PDF of Nakagami channel lead to the following results [11]

$$N_R(r) = \frac{\sqrt{2\pi} f_m}{\Gamma(m_T)} \left(\frac{m_T}{\Omega_T} r^2\right)^{m_T - \frac{1}{2}} e^{-\frac{m_T}{\Omega_T} r^2} \qquad ..(6)$$

$$\tau_R(r) = \frac{\gamma\left(m_T, \frac{m_T}{\Omega_T} r^2\right) e^{-\frac{m_T}{\Omega_T} r^2}}{\sqrt{2\pi} f_m \left(\frac{m_T}{\Omega_T} r^2\right)^{m_T - \frac{1}{2}}} \qquad ..(7)$$

where $m_T = mL, \; \Omega_T = \Omega L$.

Figure 1-2 presented the analysis of the average fade durations 'and the level crossing rates based on the maximal-ratio combining diversity in the Nakagami fading channels. Figure 1(a) compare the LCR for the diversity techniques presented above with L= 1, 2, 3, 4, 10, for m=0.6 corresponds to severe fading(worse than Reyleigh). Figure 1(b) compare the LCR above with L= 1, 2, 3, 4, 10, for m=l (Reyleigh). In the Figure 1(c), m =1.5 (approximate to Rician distribution), L= l, 2, 3, 4, 10.



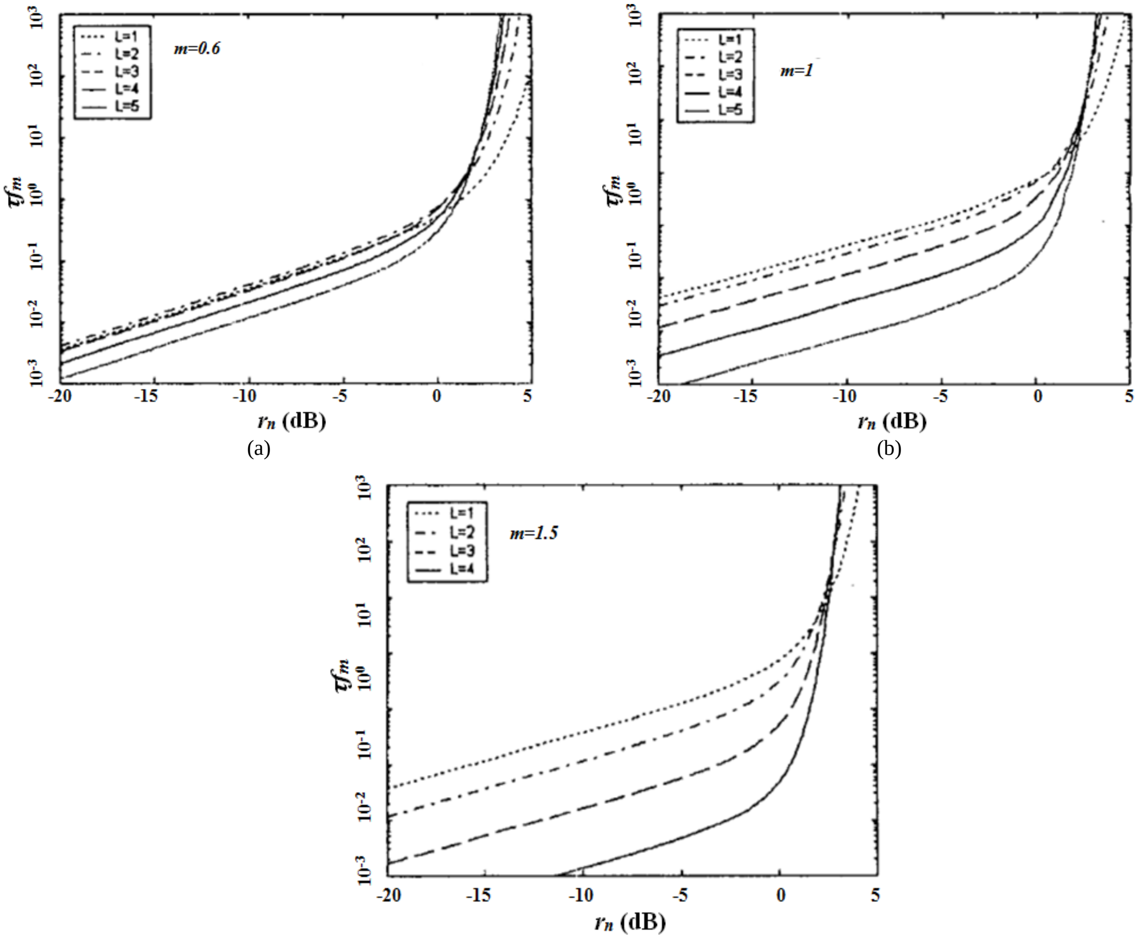
(a)

(b)

(c)

Figure.1 Normalized LCR for (a) m=0.6, (b) m=1 and (c) m=1.5 with diversity orders L=l,2,3,4,10

In figure 2(a), 2(b) & 2(c), it can be seen that with the increasing of m, if the diversity also increased, the effect of the anti-fading is more obvious.



(a) (b)

(c)

Figure 2: Normalized AFD for (a) m=0.6, (b) m=1 and (c) m=1.5 with diversity orders L=l,2,3,4,5

5.2 Capacity of MIMO channel

The capacity of the MIMO channel has been simulated for various number of transmitter and receiver antennas using the water- filling algorithm for allocation of the optimum power to the parallel sub channels, represented by the diagonal elements of the diagonal matrix which was obtained by performing the singular value decomposition on the MIMO channel matrix, Channel capacity for Nakagami faded MIMO channel is considered with implementation of waterfilling. The graph of capacity Vs SNR shows that the capacity of the MIMO channel increases as the number of antennas used at both the transmitter and receiver increases shown in figure 3.

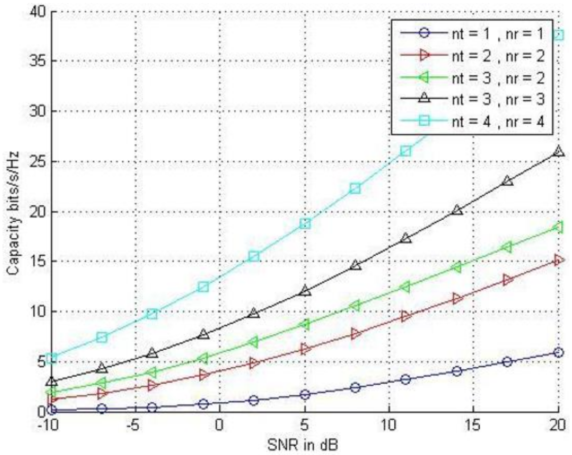


Figure 3: Channel Capacity for correlated MIMO Nakagami – m fading with Waterfilling Algorithm

VI. CONCLUSION AND FUTURE WORK

This paper simulated and analysed the statistical characteristics of Nakagami fading channels using MATLAB, as a strong software for mathematic calculation, MATLAB appears to be a simple and straightforward tool to simulate and analyse the characteristics of channels, by changes each parameters, a little change in the fading channel can be observed, it is useful for us to understand the basic conception of radio channel. This paper also gives investigations on capacity of MIMO channel. The waterfilling algorithm was implemented in this work to carry out simulations on the MIMO channel capacity and the analysis of the Nakagami – m signal fading model in wireless communication, through multipath propagation channels. The simulation in this paper can be fit to not only Namagami fading channel, but also other fading channels as well. The second-order statistics dicussed in this paper is very important in the modeling and designing of radio communication system, for example, it can be used to calculate the outage, to select the right threshold level of the receiver, to estimate the length of burst errors and so on. the paper is realistic to these engineering. In future scope, as the idea of single user power optimization in a single user MIMO system can be extended to a multi- user MIMO system.

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