Unsteady MHD Flow of two Immiscible Visco-Elastic Fluids of Higher order with Transient Pressure Gradient

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Abstract - In this paper an attempt has been made to study the flow of two immiscible visco-elastic fluids of higher order bounded by a rectilinear pipe of uniform cross section in presence of transverse magnetic field under transient pressure gradient. Towards solving the problem variable separation technique has been applied. The analytical solution of the problem has been utilised to find out the solution of the corresponding problems in the cases of visco-elastic fluids: (i) Maxwell fluids of first and second order, (ii) Oldroyd fluids of first and second order, (iii) Rivlin-Ericksen fluids of first and second order, (iv) Walters fluids and finally in case of ordinary viscous fluids also. Numerical computation of the velocity profiles have also been derived in the investigation. Clearly, these analytical solutions are very useful and it will create a new horizon in the field of fluid dynamics.

Key Words: MHD/ Visco-Elastic fluids/ Unsteady flow.

AMS Subject Classification: 76A10, 76D05, 76W05.

I. INTRODUCTION

In the field of fluid dynamics of inviscid and viscous fluids, the remarkable progress will be found in the informative books of Lamb[1], Milne-Thomson[2], Batchelor[3], Landau and Lifshitz[4]. Several hydrodynamic problems can be found in the monographs of Cowling[5], Ferraro-Plumpton[6], Cabannes[7] and Jeffrey[8]. In the area of non-Newtonian fluids the works of Bhatnagar[9] and Joseph[10] are also very worthy to mention. Bagchi[11] studied the flow of two immiscible visco-elastic Maxwell fluids under transient pressure gradient. The flow of an incompressible viscous fluid in a long rectangular channel due to a pressure gradient was studied by Drake[12]. The problem of unsteady flow of two immiscible visco-elastic fluids under a certain pressure gradient between two fixed plates was studied by Kapur and Shukla[13]. Sengupta and Raymahapatra[14] investigated the flow of two immiscible visco-elastic Maxwell fluids with transient pressure gradient through a rectangular tube. Chakraborty and Sengupta[15] studied the hydromagnetic flow of two immiscible visco-elastic Walter conducting liquid between two inclined parallel plates.

Following the above investigations and methods the authors of the present paper have investigated the unsteady MHD flow of two immiscible visco-elastic fluids of higher order under transient pressure gradient.

II. GENERAL MODEL OF VISCO ELASTIC FLUIDS

A new general model of visco-elastic fluid has been suggested by P.R. Sengupta in the following form:

$$\left.\begin{aligned} &\tau_{ij} = -p\delta_{ij} + \tau'_{ij} \\ &(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\tau'_{ij} = 2\mu(1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j})e_{ij} \\ &e_{ij} = \frac{1}{2}(v_{i,j}+v_{j,i}) \end{aligned}\right\} \quad \ldots(1)$$

Where τ_{ij} is the stress tensor, τ'_{ij} is the deviatoric stress tensor, e_{ij} is the rate of strain tensor, p is the fluid pressure, λ_j are new material constants of which the greatest value λ_1 represents the relaxation time parameter and $\lambda_2, \lambda_3, \ldots\ldots,$ λ_n are additional material constants; μ_j are also new material constants of which the greatest value μ_1 represents the strain rate retardation time parameter and μ_2, μ_3, …………., μ_n are additional material constants representing the behaviour of a very wide class of visco-elastic fluids,δ_{ij} is the metric tensor in Cartesian co-ordinates and μ the co-efficient of viscosity and v_i are the velocity components. The material constants λ_j and μ_j designating visco-elasticity satisfy the following conditions $\lambda_1>\lambda_2>\ldots\ldots\ldots>\lambda_n$ and $\mu_1>\mu_2>\ldots\ldots\ldots>\mu_n$ i.e. they are arranged in descending order of magnitudes.
Now from above:

$$\tau'_{ij} = 2\mu\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right]e_{ij} = 2\mu^* e_{ij} \text{ and } \nu^* = \frac{\mu^*}{\rho} = \frac{\mu}{\rho}\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right] = \nu\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right]$$

The fundamental Navier-Stokes equation of motion is:

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \nu^*\nabla^2\vec{q} + \vec{F} \quad \ldots (1.1)$$

Where $\vec{q}$ is velocity of the fluid, ρ is the density, p is the pressure, $\vec{F}$ is the body force vector and $\nu^* = \frac{\mu^*}{\rho}$, then (1.1) takes the form:

$$\frac{d\vec{q}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{\mu}{\rho}\left[\frac{1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j}}{1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j}}\right]\nabla^2\vec{q} + \vec{F}$$

$$i.e.\ (1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\frac{d\vec{q}}{dt} = -\frac{1}{\rho}(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\vec{\nabla}p + \nu(1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j})\nabla^2\vec{q} + (1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t^j})\vec{F}$$

Where $\nu = \frac{\mu}{\rho}$ is the kinematical co-efficient of viscosity.

III. MATHEMATICAL FORMULATION

With reference to rectangular Cartesian co-ordinate system we consider the boundary of the walls of the channel as $x = \pm a$ and $y = \pm b$. The z-axis is chosen on the surface of the fluids and towards the direction of motion of both fluids, the x-axis perpendicular to the interface drawn into the upper fluid and the y-axis in the plane of the interface.
Let$[\rho_1, \lambda_1, \lambda_2, \lambda_3, \ldots\ldots, \lambda_n\}, \{\mu_1, \mu_2, \mu_3, \ldots\ldots\ldots, \mu_n\}, \mu_L, \sigma_1]$
and$[\rho_2, \{\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3, \ldots\ldots, \bar{\lambda}_n\}, \{\bar{\mu}_1, \bar{\mu}_2, \bar{\mu}_3, \ldots\ldots, \bar{\mu}_n\}, \mu_U, \sigma_2]$ be the density, relaxation time, co-efficient of

viscosity and electrical conductivity of the lower and upper fluids respectively each occupying height 'a'.We also suppose that two media have approximately the same permeability μ_e throughout and thus the same magnetic field H_0 is interacting to both the conducting fluids,the velocities of the lower and upper fluids are respectively $w_i(x,y,t)$ [i=1,2], in the z-direction.
The equations of motion of visco-elastic conducting fluids of higher order in the presence of a transverse magnetic field in view of the above assumptions become:

$$(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t_j})\frac{\partial w_1}{\partial t} = -\frac{1}{\rho_1}(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t_j})\frac{\partial p}{\partial z} + \nu_1(1+\sum_{j=1}^{n}\mu_j \frac{\partial^j}{\partial t^j})\nabla^2 w_1 + \frac{\sigma_1 B_0{}^2}{\rho_1}(1+\sum_{j=1}^{n}\lambda_j \frac{\partial^j}{\partial t_j})w_1 \quad \ldots(2.1)$$

$$(1+\sum_{j=1}^{n}\overline{\lambda}_j \frac{\partial^j}{\partial t_j})\frac{\partial w_2}{\partial t} = -\frac{1}{\rho_2}(1+\sum_{j=1}^{n}\overline{\lambda}_j \frac{\partial^j}{\partial t_j})\frac{\partial p}{\partial z} + \nu_2(1+\sum_{j=1}^{n}\overline{\mu}_j \frac{\partial^j}{\partial t^j})\nabla^2 w_2 + \frac{\sigma_2 B_0{}^2}{\rho_2}(1+\sum_{j=1}^{n}\overline{\lambda}_j \frac{\partial^j}{\partial t_j})w_2 \quad \ldots(2.2)$$

Where $\nu_1 = \frac{\mu_L}{\rho_1}$ and $\nu_2 = \frac{\mu_U}{\rho_2}$ are the kinematical co-efficient of viscosity of the lower and upper fluids respectively and $B_0=\mu_e H_0$ is the magnetic induction vector.

IV. SOLUTION OF THE PROBLEM

Here we consider the solution of (2.1) and (2.2) as: $w_l = V_l(x)\cos my e^{-\omega t}$,[l = 1,2] …(3)

The boundary conditions of the lower fluid are:

$$\left.\begin{aligned} V_1 &= 0 \ \ when \ \ x=-a, \ -b \le y \le b \\ V_1 &= V_0 \ \ when \ \ x=0, \ \ -b \le y \le b \\ w_1 &= 0 \ \ when \ \ y=\pm b, -a \le x \le a \end{aligned}\right\} \quad \ldots(4)$$

The boundary conditions of the upper fluid are:

$$\left.\begin{aligned} V_2 &= 0 \ \ when \ \ x=+a, \ -b \le y \le b \\ V_2 &= V_0 \ \ when \ \ x=0, \ \ -b \le y \le b \\ w_2 &= 0 \ \ when \ \ y=\pm b, -a \le x \le a \end{aligned}\right\} \quad \ldots(5)$$

Equation (3) will satisfy last boundary condition of (4) and (5) and thus we have:

$$m = (2n+1)\frac{\pi}{2b} \quad \ldots(6)$$

So, all possible solution of (2.1) and (2.2) can be taken as:

$$\left.\begin{aligned} & w_l = \sum_{n=0}^{\infty} V_l(x)\cos my e^{-\omega t} \\ & \text{assuming } \frac{\partial p}{\partial z} = -Pe^{-\omega t} \end{aligned}\right\}, \ \omega > 0 \quad [l=1,2] \quad \ldots(7)$$

Using (7) from (2.1) we get:

$$\sum_{n=0}^{\infty}\frac{d^2V_1(x)}{dx^2}\cos my - \sum_{n=0}^{\infty}\left[m^2 - \frac{\omega}{\nu_1}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\} + \frac{\sigma_1 B_0{}^2}{\mu_L}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}\right]V_1(x)\cos my$$
$$+\frac{P}{\mu_L}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\} = 0 \quad \ldots(8.1)$$

Similarly for the upper fluid from (2.2) we get:

$$\sum_{n=0}^{\infty}\frac{d^2V_2(x)}{dx^2}\cos my-\sum_{n=0}^{\infty}\left[m^2-\frac{\omega}{\nu_2}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\}+\frac{\sigma_2 B_0^{\ 2}}{\mu_U}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\}\right]V_2(x)\cos my$$

$$+\frac{P}{\mu_U}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\}=0 \qquad \text{...(8.2)}$$

Now expressing $\frac{P}{\mu_U}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\}$ as well as $\frac{P}{\mu_L}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}$ as a Fourier Cosine series in the interval -b≤y≤b and equating the co-efficient of cosmy equal to zero, the values of V_1 and V_2 may be determined from:

$$\frac{d^2V_1}{dx^2}-\frac{\alpha^2}{a^2}V_1+A_n=0 \qquad \text{...(9.1)}$$

$$\frac{d^2V_2}{dx^2}-\frac{\overline{\alpha}^2}{a^2}V_1+\overline{A}_n=0 \qquad \text{...(9.2)}$$

Where $\alpha^2=\frac{\sigma_1 B_0^{\ 2}a^2}{\mu_L}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}+m^2a^2-\frac{\omega a^2}{\nu_1}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}$ ...(10)

$$A_n=\frac{(-1)^n 4P}{(2n+1)\pi\mu_L}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\} \qquad \text{...(11)}$$

$$\overline{\alpha}^2=\frac{\sigma_2 B_0^{\ 2}a^2}{\mu_U}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\}+m^2a^2-\frac{\omega a^2}{\nu_2}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\} \qquad \text{...(12)}$$

$$\overline{A}_n=\frac{(-1)^n 4P}{(2n+1)\pi\mu_U}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\} \qquad \text{...(13)}$$

Now under the boundary condition (4) we get the solution of (9.1) for the lower fluid as:

$$V_1(x)=\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}$$

Where, $-a\le x\le 0$ ...(14.1)

Again under the boundary condition (5) we get the solution of (9.2) for the upper fluid as:

$$V_2(x)=\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\overline{\alpha}^2}\left\{\frac{1+(-\omega)^n\overline{\lambda}_n}{1+(-\omega)^n\overline{\mu}_n}\right\}\left[1-\frac{\sinh\overline{\alpha}(1-\frac{x}{a})+\sinh\frac{\overline{\alpha}x}{a}}{\sinh\overline{\alpha}}\right]+V_0\frac{\sinh\overline{\alpha}(1-\frac{x}{a})}{\sinh\overline{\alpha}}$$

Where, $0\le x\le a$ ...(14.2)

Hence, for the immiscible fluids we finally get:

$$w_1=\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t}\ ;\ -a\le x\le 0 \qquad \ldots(15.1)$$

$$w_2=\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{\frac{1+(-\omega)^n\bar{\lambda}_n}{1+(-\omega)^n\bar{\mu}_n}\right\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t}\ ;\ 0\le x\le a \qquad \ldots(15.2)$$

V. INTERFACE VELOCITY, FLUX AND SKIN FRICTION

The interface velocity $w'=\sum_{n=0}^{\infty}V_0\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t}$ is obtained from the condition that the tangential stress is continuous at the interface for both fluids. Thus we get,

$$\left[\sum_{n=0}^{\infty}\mu_L\left\{\frac{1+(-\omega)^n\mu_n}{1+(-\omega)^n\lambda_n}\right\}\frac{\partial V_1}{\partial x}\cos my\right]=\left[\sum_{n=0}^{\infty}\mu_U\left\{\frac{1+(-\omega)^n\bar{\mu}_n}{1+(-\omega)^n\bar{\lambda}_n}\right\}\frac{\partial V_2}{\partial x}\cos my\right]_{x=0}$$

Using this condition we get from (14.1) and (14.2):

$$V_0=\frac{\sum_{n=0}^{\infty}\left[\frac{(-1)^n 2Pa^2}{(2n+1)\pi}\left\{\frac{\tanh\frac{\alpha}{2}}{\frac{\alpha}{2}}+\frac{\tanh\frac{\bar{\alpha}}{2}}{\frac{\bar{\alpha}}{2}}\right\}\cos(2n+1)\frac{\pi y}{2b}\right]}{\sum_{n=0}^{\infty}\left[\alpha\mu_L\coth\alpha\left\{\frac{1+(-\omega)^n\mu_n}{1+(-\omega)^n\lambda_n}\right\}+\bar{\alpha}\mu_U\coth\bar{\alpha}\left\{\frac{1+(-\omega)^n\bar{\mu}_n}{1+(-\omega)^n\bar{\lambda}_n}\right\}\right]\cos(2n+1)\frac{\pi y}{2b}} \qquad \ldots(16)$$

The total flux Q is given by:

$$Q=Q_1+Q_2=\int_{-b}^{b}\int_{-a}^{0}w_1\,dx\,dy+\int_{-b}^{b}\int_{0}^{a}w_2\,dx\,dy \qquad \ldots(17)$$

From (17) using (15.1) and (15.2) we have:

$$Q=\sum_{n=0}^{\infty}\left[\frac{16Pa^3b}{(2n+1)^2\pi^2\mu_L\alpha^2}\left\{\frac{1+(-\omega)^n\lambda_n}{1+(-\omega)^n\mu_n}\right\}\left\{1-\frac{\tanh\frac{\alpha}{2}}{\frac{\alpha}{2}}\right\}+\frac{(-1)^n 2abV_0}{(2n+1)\pi}\frac{\tanh\frac{\alpha}{2}}{\frac{\alpha}{2}}\right]e^{-\omega t}$$

$$+\sum_{n=0}^{\infty}\left[\frac{16Pa^3b}{(2n+1)^2\pi^2\mu_U\bar{\alpha}^2}\left\{\frac{1+(-\omega)^n\bar{\lambda}_n}{1+(-\omega)^n\bar{\mu}_n}\right\}\left\{1-\frac{\tanh\frac{\bar{\alpha}}{2}}{\frac{\bar{\alpha}}{2}}\right\}+\frac{(-1)^n 2abV_0}{(2n+1)\pi}\frac{\tanh\frac{\bar{\alpha}}{2}}{\frac{\bar{\alpha}}{2}}\right]e^{-\omega t} \qquad \ldots(18)$$

The skin friction on the wall x = -a is:

$$(\tau_1)_{x=-a} = \mu_L \sum_{n=0}^{\infty} \{\frac{1+(-\omega)^n \mu_n}{1+(-\omega)^n \lambda_n}\} \left[\frac{(-1)^{n+1} 4aP}{(2n+1)\pi\alpha\mu_L} \{\frac{1+(-\omega)^n \lambda_n}{1+(-\omega)^n \mu_n}\} \tanh\frac{\alpha}{2} + \frac{V_0 \alpha}{a \sinh\alpha} \right] \times$$

T

$$\cos(2n+1)\frac{\pi y}{2b} e^{-\omega t} \qquad ...(19.1)$$

he skin friction on the wall x = a is:

$$(\tau_2)_{x=a} = \mu_U \sum_{n=0}^{\infty} \{\frac{1+(-\omega)^n \bar{\mu}_n}{1+(-\omega)^n \bar{\lambda}_n}\} \left[\frac{(-1)^{n+1} 4aP}{(2n+1)\pi\bar{\alpha}\mu_U} \{\frac{1+(-\omega)^n \bar{\lambda}_n}{1+(-\omega)^n \bar{\mu}_n}\} \tanh\frac{\bar{\alpha}}{2} - \frac{V_0 \bar{\alpha}}{a \sinh\bar{\alpha}} \right] \times$$

Th

$$\cos(2n+1)\frac{\pi y}{2b} e^{-\omega t} \qquad ...(19.2)$$

e total skin friction on the wall y = b is given by:

$$(\tau)_{y=b} = (\tau_1)_{y=b} + (\tau_2)_{y=b}$$

$$= \sum_{n=0}^{\infty} \left[\begin{array}{l} \frac{2Pa^2}{b} \left\{ \frac{1}{\alpha^2} \left(1 - \frac{\sinh\alpha(1+\frac{x}{a}) - \sinh\frac{\alpha x}{a}}{\sinh\alpha} \right) + \frac{1}{\bar{\alpha}^2} \left(1 - \frac{\sinh\bar{\alpha}(1-\frac{x}{a}) + \sinh\frac{\bar{\alpha} x}{a}}{\sinh\bar{\alpha}} \right) \right\} \\ + V_0 \left\{ \frac{(-1)^n (2n+1)\pi}{2b} \left(\begin{array}{l} \frac{\mu_L \sinh\alpha(1+\frac{x}{a})}{\sinh\alpha} \times \frac{1+(-\omega)^n \mu_n}{1+(-\omega)^n \lambda_n} + \frac{\mu_U \sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}} \\ \times \frac{1+(-\omega)^n \bar{\mu}_n}{1+(-\omega)^n \bar{\lambda}_n} \end{array} \right) \right\} \end{array} \right] e^{-\omega t}$$

…(20)

Replacing b by –b in (20) we get the total friction on the wall y = -b.

VI. DEDUCTION FOR VARIOUS VISCO-ELASTIC FLUIDS

Case I: In case of two immiscible Maxwell fluids we take λ_1, $\bar{\lambda}_1 > 0$ and $\lambda_j = \bar{\lambda}_j = 0$ (j=2,3,…..,n); $\mu_j = 0 = \bar{\mu}_j$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty} \left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L \alpha^2} \{1-\lambda_1\omega\} \left\{ 1 - \frac{\sinh\alpha(1+\frac{x}{a}) - \sinh\frac{\alpha x}{a}}{\sinh\alpha} \right\} + V_0 \frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha} \right] \times$$

$$\cos(2n+1)\frac{\pi y}{2b} e^{-\omega t} \quad ; \ -a \le x \le 0 \qquad …(21.1)$$

$$w_2 = \sum_{n=0}^{\infty} \left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U \bar{\alpha}^2} \{1-\bar{\lambda}_1\omega\} \left\{ 1 - \frac{\sinh\bar{\alpha}(1-\frac{x}{a}) + \sinh\frac{\bar{\alpha} x}{a}}{\sinh\bar{\alpha}} \right\} + V_0 \frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}} \right] \times$$

$$\cos(2n+1)\frac{\pi y}{2b} e^{-\omega t} \quad ; \ 0 \le x \le a \qquad …(21.2)$$

Where $\alpha^2 = \frac{\sigma_1 B_0^{\,2} a^2}{\mu_L}\{1-\lambda_1\omega\} + m^2 a^2 - \frac{\omega a^2}{\nu_1}\{1-\lambda_1\omega\}$

And $\bar{\alpha}^2 = \frac{\sigma_2 B_0^{\,2} a^2}{\mu_U}\{1-\bar{\lambda}_1\omega\} + m^2 a^2 - \frac{\omega a^2}{\nu_2}\{1-\bar{\lambda}_1\omega\}$

Case II: In case of two immiscible Maxwell fluids of second order we take $\lambda_1>\lambda_2>0$, $\bar{\lambda}_1>\bar{\lambda}_2>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=3,4…..,n); $\mu_j=0=\bar{\mu}_j$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\{1-\lambda_1\omega+\lambda_2\omega^2\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a \le x \le 0 \qquad \dots(22.1)$$

$$w_2 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\{1-\bar{\lambda}_1\omega+\bar{\lambda}_2\omega^2\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0 \le x \le a \qquad \dots(22.2)$$

Where $\alpha^2 = \frac{\sigma_1 B_0^{\,2} a^2}{\mu_L}\{1-\lambda_1\omega+\lambda_2\omega^2\} + m^2 a^2 - \frac{\omega a^2}{\nu_1}\{1-\lambda_1\omega+\lambda_2\omega^2\}$

And $\bar{\alpha}^2 = \frac{\sigma_2 B_0^{\,2} a^2}{\mu_U}\{1-\bar{\lambda}_1\omega+\bar{\lambda}_2\omega^2\} + m^2 a^2 - \frac{\omega a^2}{\nu_2}\{1-\bar{\lambda}_1\omega+\bar{\lambda}_2\omega^2\}$

Case III: In case of two immiscible Oldroyd fluids we take λ_1, $\bar{\lambda}_1$, μ_1, $\bar{\mu}_1>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=2,3,…..,n); $\mu_j=0=\bar{\mu}_j$ (j=2,3,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1-\lambda_1\omega}{1-\mu_1\omega}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a \le x \le 0 \qquad \dots(23.1)$$

$$w_2 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{\frac{1-\bar{\lambda}_1\omega}{1-\bar{\mu}_1\omega}\right\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0 \le x \le a \qquad \dots(23.2)$$

Where $\alpha^2 = \frac{\sigma_1 B_0^{\,2} a^2}{\mu_L}\left\{\frac{1-\lambda_1\omega}{1-\mu_1\omega}\right\} + m^2 a^2 - \frac{\omega a^2}{\nu_1}\left\{\frac{1-\lambda_1\omega}{1-\mu_1\omega}\right\}$

And $\bar{\alpha}^2 = \frac{\sigma_2 {B_0}^2 a^2}{\mu_U}\left\{\frac{1-\bar{\lambda}_1\omega}{1-\bar{\mu}_1\omega}\right\} + m^2a^2 - \frac{\omega\, a^2}{\nu_2}\left\{\frac{1-\bar{\lambda}_1\omega}{1-\bar{\mu}\omega}\right\}$

Case IV: In case of two immiscible Oldroyd fluids of second order we take $\lambda_1>\lambda_2>0$, $\bar{\lambda}_1>\bar{\lambda}_2>0$; $\mu_1>\mu_2>0$, $\bar{\mu}_1>\bar{\mu}_2>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=3,4…..,n); $\mu_j=0=\bar{\mu}_j$ (j=3,4,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1-\lambda_1\omega+\lambda_2\omega^2}{1-\mu_1\omega+\mu_2\omega^2}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a \le x \le 0 \qquad \text{…(24.1)}$$

$$w_2 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{\frac{1-\bar{\lambda}_1\omega+\bar{\lambda}_2\omega^2}{1-\bar{\mu}_1\omega+\bar{\mu}_2\omega^2}\right\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0 \le x \le a \qquad \text{…(24.2)}$$

Where $\alpha^2 = \frac{\sigma_1 {B_0}^2 a^2}{\mu_L}\left\{\frac{1-\lambda_1\omega+\lambda_2\omega^2}{1-\mu_1\omega+\mu_2\omega^2}\right\} + m^2a^2 - \frac{\omega\, a^2}{\nu_1}\left\{\frac{1-\lambda_1\omega+\lambda_2\omega^2}{1-\mu_1\omega+\mu_2\omega^2}\right\}$

And $\bar{\alpha}^2 = \frac{\sigma_2 {B_0}^2 a^2}{\mu_U}\left\{\frac{1-\bar{\lambda}_1\omega+\bar{\lambda}_2\omega^2}{1-\bar{\mu}_1\omega+\bar{\mu}_2\omega^2}\right\} + m^2a^2 - \frac{\omega\, a^2}{\nu_2}\left\{\frac{1-\bar{\lambda}_1\omega+\bar{\lambda}_2\omega^2}{1-\bar{\mu}_1\omega+\bar{\mu}_2\omega^2}\right\}$

Case V: In case of two immiscible Rivlin-Ericksen fluids we take μ_1, $\bar{\mu}_1>0$ and $\mu_j=\bar{\mu}_j=0$ (j=2,3,…..,n); $\lambda_j=0=\bar{\lambda}_j$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1}{1-\mu_1\omega}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a \le x \le 0 \qquad \text{…(25.1)}$$

$$w_2 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{\frac{1}{1-\bar{\mu}_1\omega}\right\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0 \le x \le a \qquad \text{…(25.2)}$$

Where $\alpha^2 = \frac{\sigma_1 {B_0}^2 a^2}{\mu_L}\left\{\frac{1}{1-\mu_1\omega}\right\} + m^2a^2 - \frac{\omega\, a^2}{\nu_1}\left\{\frac{1}{1-\mu_1\omega}\right\}$

And $\bar{\alpha}^2 = \frac{\sigma_2 {B_0}^2 a^2}{\mu_U}\left\{\frac{1}{1-\bar{\mu}_1\omega}\right\} + m^2a^2 - \frac{\omega\, a^2}{\nu_2}\left\{\frac{1}{1-\bar{\mu}_1\omega}\right\}$

Case VI: In case of two immiscible Rivlin-Ericksen fluids of second order we take $\mu_1>\mu_2>0$, $\bar{\mu}_1>\bar{\mu}_2>0$ and $\lambda_j=\bar{\lambda}_j=0$ (j=1,2,…..,n); $\mu_j=0=\bar{\mu}_j$ (j=3,4,…..,n).So the velocities of the lower and upper fluids are:

$$w_1=\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1}{1-\mu_1\omega+\mu_2\omega^2}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a\le x\le 0 \qquad \text{…(26.1)}$$

$$w_2=\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{\frac{1}{1-\bar{\mu}_1\omega+\bar{\mu}_2\omega^2}\right\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0\le x\le a \qquad \text{…(26.2)}$$

Where $\alpha^2=\frac{\sigma_1 B_0^2 a^2}{\mu_L}\left\{\frac{1}{1-\mu_1\omega+\mu_2\omega^2}\right\}+m^2a^2-\frac{\omega a^2}{\nu_1}\left\{\frac{1}{1-\mu_1\omega+\mu_2\omega^2}\right\}$

And $\bar{\alpha}^2=\frac{\sigma_2 B_0^2 a^2}{\mu_U}\left\{\frac{1}{1-\bar{\mu}_1\omega+\bar{\mu}_2\omega^2}\right\}+m^2a^2-\frac{\omega a^2}{\nu_2}\left\{\frac{1}{1-\bar{\mu}_1\omega+\bar{\mu}_2\omega^2}\right\}$

Case VII: In case of two immiscible Walters fluids we take $\mu_1=-\beta$, $\bar{\mu}_1=-\bar{\beta}$ (β, $\bar{\beta}>0$) and $\lambda_j=\bar{\lambda}_j=0$ (j=1,2,3,…..,n); $\mu_j=0=\bar{\mu}_j$ (j=2,3,…..,n).So the velocities of the lower and upper fluids are:

$$w_1=\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{\frac{1}{1+\beta\omega}\right\}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a\le x\le 0 \qquad \text{…(27.1)}$$

$$w_2=\sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{\frac{1}{1+\bar{\beta}\omega}\right\}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0\le x\le a \qquad \text{…(27.2)}$$

Where $\alpha^2=\frac{\sigma_1 B_0^2 a^2}{\mu_L}\left\{\frac{1}{1+\beta\omega}\right\}+m^2a^2-\frac{\omega a^2}{\nu_1}\left\{\frac{1}{1+\beta\omega}\right\}$

And $\bar{\alpha}^2=\frac{\sigma_2 B_0^2 a^2}{\mu_U}\left\{\frac{1}{1+\bar{\beta}\omega}\right\}+m^2a^2-\frac{\omega a^2}{\nu_2}\left\{\frac{1}{1+\bar{\beta}\omega}\right\}$

Case VIII: In case of two immiscible purely viscous fluids we take $\lambda_j\to 0$, $\bar{\lambda}_j\to 0$; $\mu_j\to 0$, $\bar{\mu}_j\to 0$ (j=1,2,…..,n).So the velocities of the lower and upper fluids are:

$$w_1 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_L\alpha^2}\left\{1-\frac{\sinh\alpha(1+\frac{x}{a})-\sinh\frac{\alpha x}{a}}{\sinh\alpha}\right\}+V_0\frac{\sinh\alpha(1+\frac{x}{a})}{\sinh\alpha}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; -a \le x \le 0 \qquad \ldots(28.1)$$

$$w_2 = \sum_{n=0}^{\infty}\left[\frac{(-1)^n 4Pa^2}{(2n+1)\pi\mu_U\bar{\alpha}^2}\left\{1-\frac{\sinh\bar{\alpha}(1-\frac{x}{a})+\sinh\frac{\bar{\alpha}x}{a}}{\sinh\bar{\alpha}}\right\}+V_0\frac{\sinh\bar{\alpha}(1-\frac{x}{a})}{\sinh\bar{\alpha}}\right]\times$$

$$\cos(2n+1)\frac{\pi y}{2b}e^{-\omega t} \quad ; 0 \le x \le a \qquad \ldots(28.2)$$

Where $\alpha^2 = \dfrac{\sigma_1 B_0{}^2 a^2}{\mu_L} + m^2a^2 - \dfrac{\omega a^2}{\nu_1}$

And $\bar{\alpha}^2 = \dfrac{\sigma_2 B_0{}^2 a^2}{\mu_U} + m^2a^2 - \dfrac{\omega a^2}{\nu_2}$

VII. NUMERICAL CALCULATIONS AND DISCUSSIONS

The analytical results for velocity as exhibited in the earlier articles has been numerically computed for different times as in the following tables and graphs. For this we take λ_1=.04, μ_1=.02, μ_L=.06, ν_1=.04, a=.5, b=.25, x=-.1(lower fluid),y=.2, $M_1^2 = \sigma_1 B_0^2 a^2/\mu_L = M_2^2 = \sigma_2 B_0^2 a^2/\mu_U$ =1, V_0=5, ω=.1, $\bar{\lambda}_1$=.06, $\bar{\mu}_1$=.03, μ_U=.05, ν_2=.03, x=.1(upper fluid).

From the following tables and graphs it is quite clear that in the cases of two immiscible visco-elastic Maxwell, Oldroyd and Rivlin -Ericksen fluids as well as viscous fluid, the velocities decrease continually with time for both the fluids. We also find that the velocity of a fluid element starts with a maximum velocity and dies out asymptotically with the increase of time. This feature of investigation of the titled problem is perfectly in conformity with the usual concept of magneto-hydrodynamics. In case of purely viscous fluid or visco-elastic fluids the velocity profile has a maximum value in absence of the magnetic field.

In the following four tables the velocities of two immiscible fluids in case of visco-elastic Oldroyd fluids, Maxwell fluids, Rivlin-Ericksen fluid and ordinary viscous fluid are presented, respectively.

OLDROYD FLUIDS **MAXWELL FLUIDS**

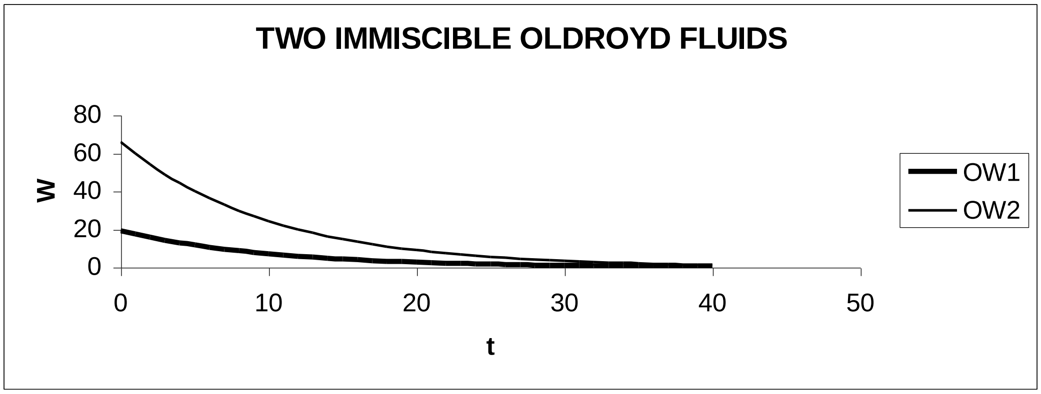
t	OW1	OW2
0	19.2365	65.99734
1	17.40590499	59.71686272
2	15.74951413	54.03405188
3	14.2507497	48.89203199
4	12.89461157	44.23933999
5	11.66752704	40.02941017
6	10.55721504	36.22010814
7	9.552563196	32.77330913
8	8.643516618	29.65451642
9	7.82097726	26.83251607
10	7.07671287	24.27906456
11	6.403274602	21.96860609
12	5.793922457	19.87801681
13	5.242557837	17.98637341
14	4.743662497	16.27474367
15	4.292243326	14.72599704
16	3.883782368	13.32463314
17	3.51419161	12.05662665
18	3.179772063	10.90928693
19	2.877176744	9.871131016
20	2.603377176	8.931768702
21	2.355633082	8.081798531
22	2.131464956	7.312713716
23	1.928629247	6.616816997
24	1.745095908	5.987143607
25	1.579028076	5.417391563
26	1.428763687	4.901858594
27	1.292798846	4.435385074
28	1.16977277	4.013302378
29	1.058454173	3.631386162
30	0.957728941	3.285814079
31	0.866588982	2.973127527
32	0.784122137	2.690197035
33	0.70950305	2.434190939
34	0.641984908	2.202547044
35	0.580891966	1.992946981
36	0.525612787	1.803293
37	0.475594117	1.631686982
38	0.430335353	1.476411436
39	0.38938353	1.335912312
40	0.352328787	1.208783447

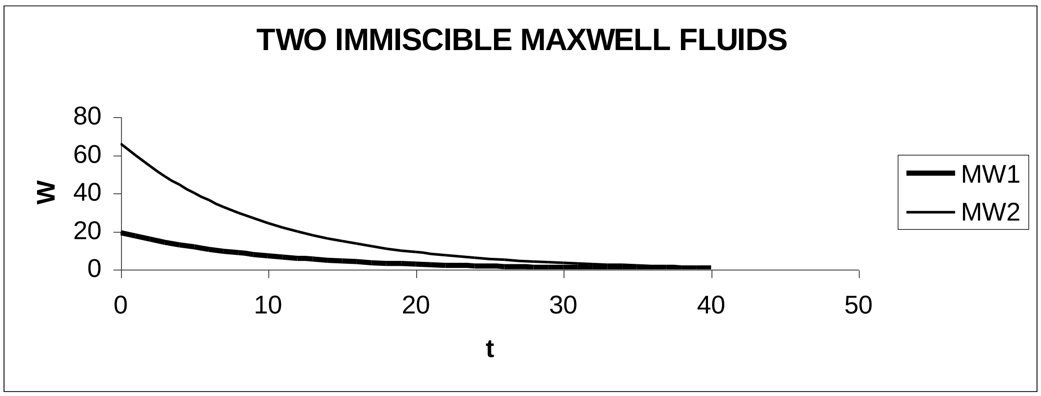
t	MW1	MW2
0	19.24725	65.982
1	17.41563199	59.70298252
2	15.75831549	54.02149255
3	14.2587135	48.88066784
4	12.90181751	44.22905728
5	11.67404724	40.02010599
6	10.56311476	36.21168937
7	9.557901488	32.76569151
8	8.648346905	29.64762371
9	7.825347883	26.82627929
10	7.080667574	24.27342129
11	6.406852966	21.96349984
12	5.797160295	19.87339649
13	5.245487553	17.98219277
14	4.746313414	16.27096087
15	4.294641975	14.72257423
16	3.885952756	13.32153605
17	3.516155458	12.05382428
18	3.181549026	10.90675124
19	2.878784606	9.868836634
20	2.60483203	8.929692659
21	2.356949489	8.079920049
22	2.13265609	7.311013995
23	1.92970703	6.615279027
24	1.746071126	5.985751994
25	1.57991049	5.416132379
26	1.429562128	4.900719238
27	1.293521305	4.434354142
28	1.170426478	4.012369552
29	1.059045672	3.630542106
30	0.958264152	3.285050345
31	0.867073261	2.972436472
32	0.784560331	2.689571743
33	0.709899544	2.433625151
34	0.64234367	2.202035099
35	0.581216588	1.992483753
36	0.525906517	1.802873855
37	0.475859895	1.631307724
38	0.430575839	1.476068269
39	0.38960113	1.335601801
40	0.352525681	1.208502485

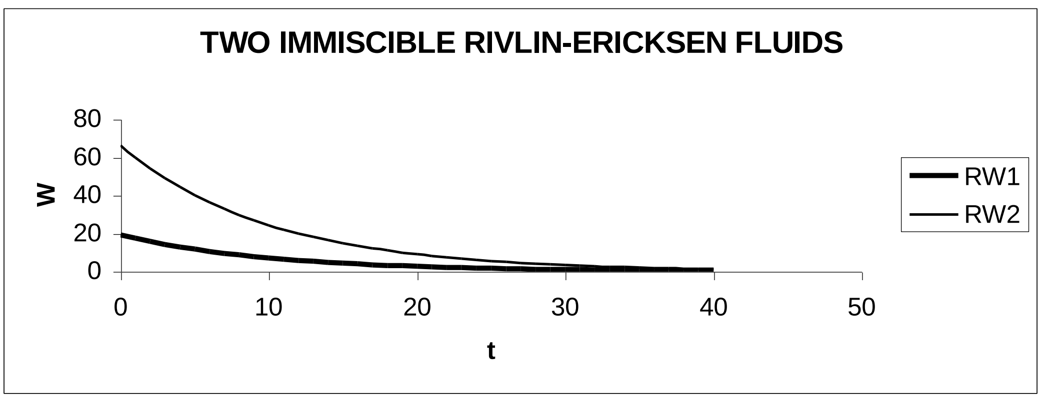
RIVLIN-ERICKSEN FLUIDS **ORDINARY VISCOUS FLUIDS**

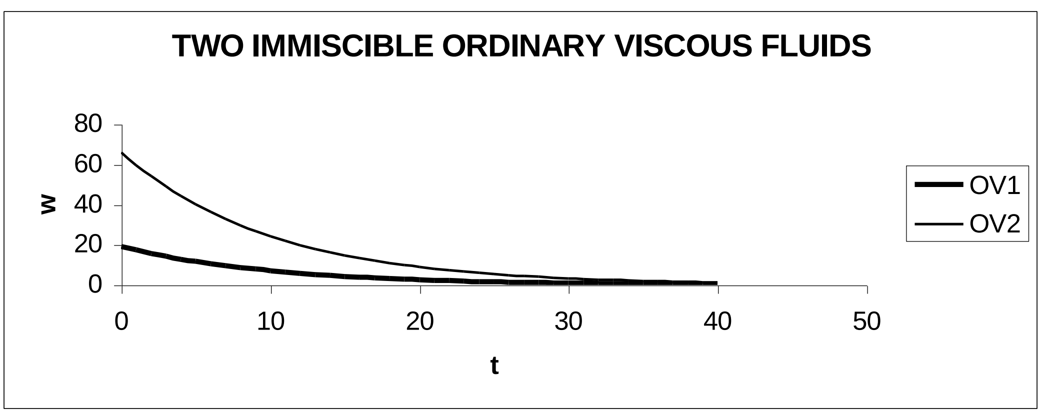
t	RW1	RW2
0	19.21499	66.12085
1	17.38644194	59.82861919
2	15.73190323	54.13517331
3	14.2348147	48.98353045
4	12.88019298	44.32213122
5	11.65448056	40.10432277
6	10.5454101	36.28789187
7	9.541881646	32.83464238
8	8.633851552	29.71001304
9	7.812231946	26.88273149
10	7.068799783	24.32450135
11	6.396114545	22.00971899
12	5.78744377	19.91521731
13	5.236695678	18.02003381
14	4.738358196	16.30520086
15	4.287443796	14.75355585
16	3.879439574	13.34956938
17	3.510262088	12.07918989
18	3.176216484	10.92970299
19	2.873959523	9.889604236
20	2.600466114	8.948483963
21	2.352999044	8.096923124
22	2.12908158	7.326399014
23	1.92647268	6.629199967
24	1.743144565	5.998348182
25	1.577262428	5.427529881
26	1.427166063	4.911032124
27	1.291353255	4.443685627
28	1.168464745	4.020813029
29	1.057270623	3.63818208
30	0.956658021	3.291963279
31	0.865619974	2.978691554
32	0.783245342	2.695231575
33	0.708709693	2.438746379
34	0.641267049	2.206668977
35	0.58024242	1.99667666
36	0.525025054	1.806667753
37	0.475062314	1.634740585
38	0.429854158	1.47917445
39	0.388948126	1.33841239
40	0.351934818	1.211045612

t	OV1	OV2
0	19.22574	66.04377
1	17.39616894	59.75887432
2	15.74070459	54.07206555
3	14.2427785	48.92642818
4	12.88739892	44.27046295
5	11.66100077	40.05757139
6	10.55130982	36.24558947
7	9.547219939	32.79636559
8	8.638681839	29.67537876
9	7.81660257	26.8513931
10	7.072754487	24.2961452
11	6.399692909	21.98406129
12	5.790681608	19.89200126
13	5.239625395	17.99902706
14	4.741009114	16.28619317
15	4.289842445	14.73635698
16	3.881609962	13.3340072
17	3.512225936	12.06510865
18	3.177993447	10.91696175
19	2.875567385	9.878075487
20	2.601920968	8.938052319
21	2.354315451	8.087484183
22	2.130272714	7.317858306
23	1.927550462	6.621472015
24	1.744119783	5.991355642
25	1.578144841	5.42120277
26	1.427964504	4.905307117
27	1.292075715	4.438505426
28	1.169118453	4.01612579
29	1.057862123	3.63394089
30	0.957193232	3.288125692
31	0.866104252	2.975219162
32	0.783683536	2.692089624
33	0.709106187	2.435903425
34	0.641625811	2.204096565
35	0.580567042	1.994349045
36	0.525318784	1.804561641
37	0.475328092	1.632834896
38	0.430094643	1.477450111
39	0.389165727	1.336852144
40	0.352131711	1.209633842


TWO IMMISCIBLE OLDROYD FLUIDS
W
t
0
20
40
60
80
0
10
20
30
40
50
OW1
OW2


TWO IMMISCIBLE MAXWELL FLUIDS
W
t
0
20
40
60
80
0
10
20
30
40
50
MW1
MW2


TWO IMMISCIBLE RIVLIN-ERICKSEN FLUIDS
W
t
0
20
40
60
80
0
10
20
30
40
50
RW1
RW2


TWO IMMISCIBLE ORDINARY VISCOUS FLUIDS
w
t
0
20
40
60
80
0
10
20
30
40
50
OV1
OV2

REFERENCES

[1] Lamb, H. - Hydrodynamics, Dover Publications, New York, (1945)
[2] Milne-Thomson, L.M – Theoretical hydrodynamics, New York, The Macmilan Company (1955).
[3] Batchelor, G.K.- An introduction to fluid dynamics, Cambridge University Press (1967).
[4] Landau, I.D. and Lifshitz, E.M. – Fluid mechanics, New York, Pergamon Press (1959).
[5] Cowling, T.G. – Magneto-hydrodynamics, Inter-Science, Pub. Inc. New York (1957).
[6] Ferraro. V.C.A and Plumpton, C. – Magneto fluid mechanics, Larendon Press, Oxford,London (1966).
[7] Cabannes , H. – Theoretical magneto fluid dynamics, Academic Press, New York and London (1970).
[8] Jeffrey, A. – Magneto hydrodynamics, Oliver and Boyd, Edinburgh and London, New York, Interscience Publishers Inc., A division of John Wiley and Sons, Inc. (1966).
[9] Bhatnagar, P.L – The summer seminar in fluid mechanics, Department of Applied Mathematics, Indian Institute of Science, Bangalore-12, India, May (1967).
[10] Joseph, D.D – Fluid dynamics of visco-elastic liquids, University of Minnesota Minneapolis, M.N Springer-Verlag, London (1990).
[11] Bagchi, K.C. (1965); Ind. Jour. Mech. Math, (IV), 2, 87.
[12] Drake, D.H.-Quart. Jour. Mech. Applied Math., Vol. 18, No.1, p. 1 (1965).
[13] Kapur, J.N. and Shukla, J.B. (1962); ZAMM, 44, 6, 268.
[14] Sengupta, P.R. and Raymahapatra, J. (1971); Rev. Roum. Sci. Tech. Mech. Appl. Tome. 39, No. 6, 635.
[15] Chakraborty, G. and Sengupta, P.R.(1994); Rev. Roum. Sci. Techn. Mech. Appl. Tome, Bucharest, 39, 6, 635.