Effect of Radation on Unsteady MHD Flow through Porous Medium and Mass and Heat Transfer Past A Porous Vertical Moving Plate

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Abstract- An exact solution to the problem on unsteady MHD free convection flow of an incompressible, viscous, electrically conducting fluid through porous media past an infinite porous vertical non conducting moving plate in the presence of uniform transverse magnetic field is investigated. This type of problem finds application in many technological and engineering fields such as rocket propulsion systems, space craft re-entry aerothermodynamics, cosmical flight aerodynamics, plasma physics, Glass production and furnace engineering. The Roselland approximation is used to describe the radiative heat transfer in the limit of the optically thin fluid. Velocity, temperature and concentration of the flow have been presented for various parameters such as thermal Grashof number, Prandtl number, Schmidt number, radiation parameter and magnetic parameter. The local and average skin fraction, Nusslet numbers are also presented.

Keywords: Radiation, heat and mass transfer, MHD.

I. INTRODUCTION

Thermal boundary layer flow problems are classified into two categories e.g., (i) free/natural convection flow and (ii) forced convection flow and have many applications in the areas of industries and engineering. The Grashof number, Prandtl number, Hertmann number and porosity play an important role on free convection flows. The problem of free convection flows past a porous/non porous vertical plate has been considered by many researchers, i.g., Schlichting [7], Gupta [3], Soundalgekar [10], Mishra and Mohapatra [4]. Soundalgekar and Gupta [11], Bansal [1], Soundalgekar [12], Georgantopoulos [2], Rapits and Tzivanidis [6] Mohapatra and Senpati [5], Sharma [8], and Sharma and Mishra [9] etc.

In the context of space technology and in processes involving high temperatures the effects of radiation are of vital importance. Recent developments in hypersonic flights, missile reentry, rocket combustion chambers, and power plants for inter planetary flight and gas cooled nuclear reactors, have focused attention on thermal radiation as a mode of radiative transfer in these process. The interaction of radiation with laminar free convection heat transfer from a vertical plat was investigate by Cess [14] for an absorbing, emitting fluid in the optically thick region, using the singular perturbation technique. Arpaci [15] considered a similar problem in both the optically thick regions and used the approximate integral technique and first-order profiles to solve the energy equation. Cheng and Ozisik [16] considered a related problem for an absorbing, emitting and isotropically scattering fluid, and treated the radiation part of the problem exactly with the normal – mode expansion technique. Raptis [17] has analyzed the thermal radiation and free convection flow through a porous medium by using perturbation technique. Hossain and Takhar [18] studied the radiation effects on mixed convection along a vertical plate with uniform surface temperature using Keller Box finite difference method. In all these papers the flow is considered to be study. The unsteady flow past a moving plate in the presence of free convection and radiation were studied by Mansour [19]. Raptis and Perdikis [20] studied the effects of thermal radiation and free convective flow past moving plate. Das et al. [21] have analyzed the radiation effects on flow past an impulsively started infinite isothermal vertical plate. Chamkha et al [22] studied the effect of radiation on free convective flow past a semi-infinite vertical plate with mass transfer. Genesan and Loganadhan [23] studied the radiation and mass transfer effects on flow of incompressible viscous fluid past moving vertical cylinder using Rosseland approximation.

In the present study we consider the problem Kumar et al [13] with radiation. The purpose of this study is to investigate the effect of radiation on unsteady MHD free convection flow an incompressible viscous, electrically conducting fluid through porous media past an infinite porous vertical non-conducting moving plate in the presence of uniform magnetic field.

II. FORMULATION OF THE PROBLEM

We consider a hot vertical porous infinite moving plate with constant velocity in upward direction i.e., along x-axis and y-axis is taken normal to it and a uniform transverse magnetic field is applied. The fluid is withdrawn through the plate at a constant rate. The governing equations of motion and energy for the unsteady flow of an incompressible viscous conducting fluid along a hot vertical plate in the presence of transverse magnetic field with Boussinesq's approximation the unsteady flow past the infinite moving plate is governed by the following equations.

$$\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{dU(t)}{dt}+GrT+GcC+\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)(u-U(t)) \tag{1}$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_P}\frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where u is the velocity component along x-axis, y-axis normal to x - axis, t is the time, T is the temperature, C is the concentration, Gr is the Grashof number, Gc is the modified Grashof number, M is the Hartmann number, K is the porosity parameter, Pr is the Prandtl number and Sc is the Schmidt number, CP is the specific heat at a constant pressure, ρ is the density. qr radiative heat flux.

The boundary conditions in non-dimensional form are

$$\left.\begin{array}{llll} y=0 : & u=1, & T=1, & C=1 \\ y\to\infty : & u\to U(t), & T\to 0, & C\to 0 \end{array}\right\} \tag{4}$$

where U (t) is the free stream velocity.

We now assume Rosseland approximation, which leads to the radiative heat flux qr is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzmann constant and K^* is the mean absorption coefficient.

If the temperature differences within the flow are sufficiently small such that T4 may be expressed as a linear function of the temperature, then the Taylor series for T4 above T_∞, after neglecting higher order terms, is given by

$$T^4 \cong 4T^3_\infty T-3T^4_\infty \tag{6}$$

In view of Eqs (5) and (6), Eqs (2) reduces to

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T^3}{3k^*\rho C_P}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

or

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{N_1}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

where

$$N_1=\frac{3NP_r}{3N+4}$$

$$N = \left(\frac{k^* K}{4\sigma^* T^3_\infty} \right) \text{ (Radiation parameter)}$$

III. METHOD OF SOLUTION

For separation into mean flow and unsteady flow, we assume

$$\left.\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y) e^{i\omega t} \\ T(y,t) &= T_0(y) + \varepsilon T_1(y) e^{i\omega t} \\ C(y,t) &= C_0(y) + \varepsilon C_1(y) e^{i\omega t} \\ U(t) &= 1 + \varepsilon e^{i\omega t} \end{aligned}\right\} \tag{9}$$

using into the equations (1), (3) and (8) and equating the like powers of O (ε), we get

ZEROTH ORDER:

$$u_0'' + u_0' - \left(M + \frac{1}{K} \right) u_0 = -GrT_0 - GcC_0 - \left(M + \frac{1}{K} \right) \tag{10}$$

$$T_0'' + N_1 T_0' = 0 \tag{11}$$

$$C_0'' + S_c C_0' = 0 \tag{12}$$

FIRST ORDER:

$$u_1'' + u_1' - \left(M + \frac{1}{K} + i\omega \right) u_1 = -GrT_1 - GcC_1 - \left(M + \frac{1}{K} + i\omega \right) \tag{13}$$

$$C_1'' + ScC_1' - i\omega\, ScC_1 = 0 \tag{14}$$

$$T_1'' + N_1 T_1' - i\omega\, N_1 T_1 = 0 \tag{15}$$

where prime denotes differentiation to 'y'.

Now the corresponding boundary conditions are

$$\left.\begin{aligned} y = 0 :\quad & u_0 = 1,\quad u_1 = 0,\quad T_0 = 1,\quad T_1 = 0,\quad C_0 = 1,\quad C_1 = 0 \\ y = \infty :\quad & u_0 = 1,\quad u_1 = 1,\quad T_0 = 0,\quad T_1 = 0,\quad C_0 = 0,\quad C_1 = 0 \end{aligned}\right\} \tag{16}$$

The equations (11), (12), (14) and (15) are ordinary linear differential equations and solve under the boundary condition (16). Thus we get

$$T_0(y) = e^{-N_1 y} \qquad T_1(y) = 0 \tag{17}$$

$$C_0(y) = e^{-Scy} \qquad C_1(y) = 0 \tag{18}$$

Using equations (17) and (18) in the equations (10) and (11), solutions of these equations are given by

$$u_0(y) = 1 + (L_1 Gr + L_2 Gc) e^{-A_1 y} - L_1 Gr e^{-N_1 y} - L_2 Gc e^{-Scy} \tag{19}$$

$$u_1(y) = F_1(y) + iF_2(y) \tag{20}$$

Finally, the expressions of u (y, t), T (y, t) and C (y, t) are obtained as:

$$u(y,t) = u_0(y) + \varepsilon(F_1\cos\omega t - F_2\sin\omega t) \tag{21}$$

$$T_0(y) = e^{-N_1 y} \tag{22}$$

$$C_0(y) = e^{-Scy} \tag{23}$$

SKIN-FRICTION:

The skin-friction co-efficient at the plate is given by

$$Cf = \frac{\tau}{\rho U_0 \nu_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= -A_1 (L_1 Gr + L_2 Gc) + L_1 N_1 Gr + L_2 Sc\, Gc + \varepsilon(A_2 \cos \omega t - B_2 \sin \omega t) \tag{24}$$

NUSSELT NUMBER

The rate of heat transfer in the terms of the Nusselt number at the plate is given by

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = N_1$$

where $A_1 = \dfrac{1+\sqrt{1+\left(M+\frac{1}{K}\right)}}{2}$, $A_2 = \dfrac{1+\alpha_1}{2}$, $B_2 = \dfrac{\beta_2}{2}$, $F_1(y) = 1 - e^{-A_2 y}\cos B_2 y$, $F_2(y) = e^{-A_2 y}\sin B_2 y$

.

$$L_1 = \frac{1}{\left[\Pr^2 - \Pr - \left(M+\frac{1}{K}\right)\right]}, \qquad L_2 = \frac{1}{\left[Sc^2 - Sc - \left(M+\frac{1}{K}\right)\right]}$$

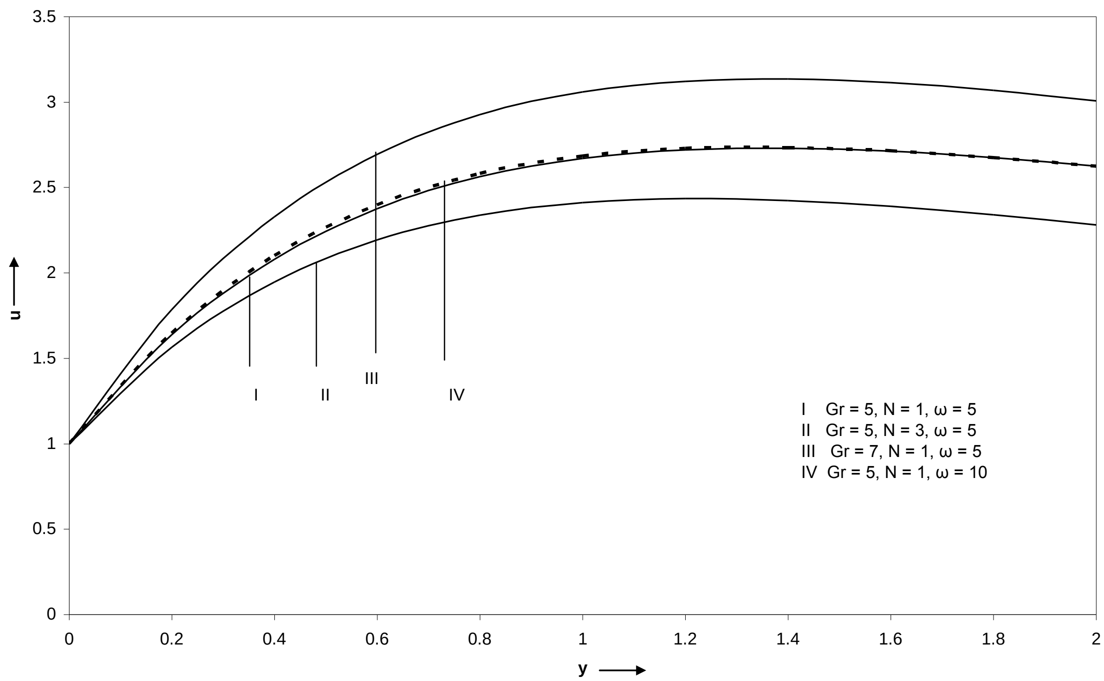
$$\alpha_1 = \left[\frac{\sqrt{\left\{1+4\left(M+\frac{1}{K}\right)\right\}^2 + 16\omega^2} + \left\{1+4\left(M+\frac{1}{K}\right)\right\}}{2}\right]^{1/2}$$

$$\beta_1 = \left[\frac{\sqrt{\left\{1+4\left(M+\frac{1}{K}\right)\right\}^2 + 16\omega^2} - \left\{1+4\left(M+\frac{1}{K}\right)\right\}}{2}\right]^{1/2}$$

IV. RESULT AND DISCUSSIONS

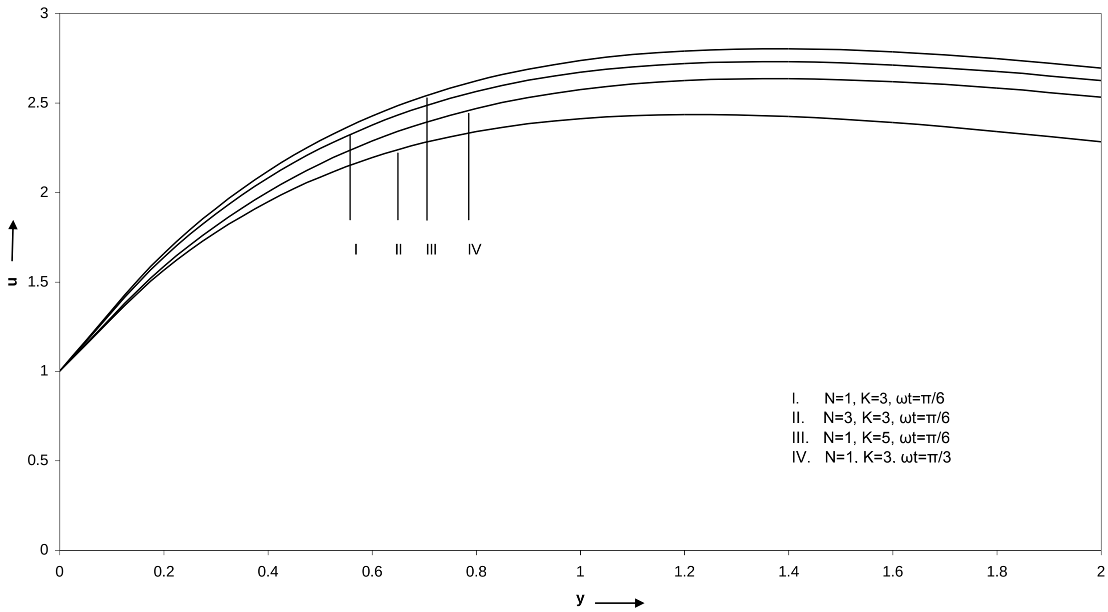
In figure 1, the velocity distribution of boundary layer flow is plotted against y for different ε = .25, M = 2, K = 3, Pr = 0.71, Sc = .4, Gc = 3, and ωt = π/6 and different values of N, Gr, ω. It is observed that the fluid velocity increases till y = 1.2, after its velocity decreases continuously with increases in y. It is observed that the fluid velocity decreases due to increasing the radiation parameter N. However, fluid velocity increases due to increasing Grashof number Gr, frequency parameter ω.

Fig.1. The velocity distribution for different values of Gr, N, and ω



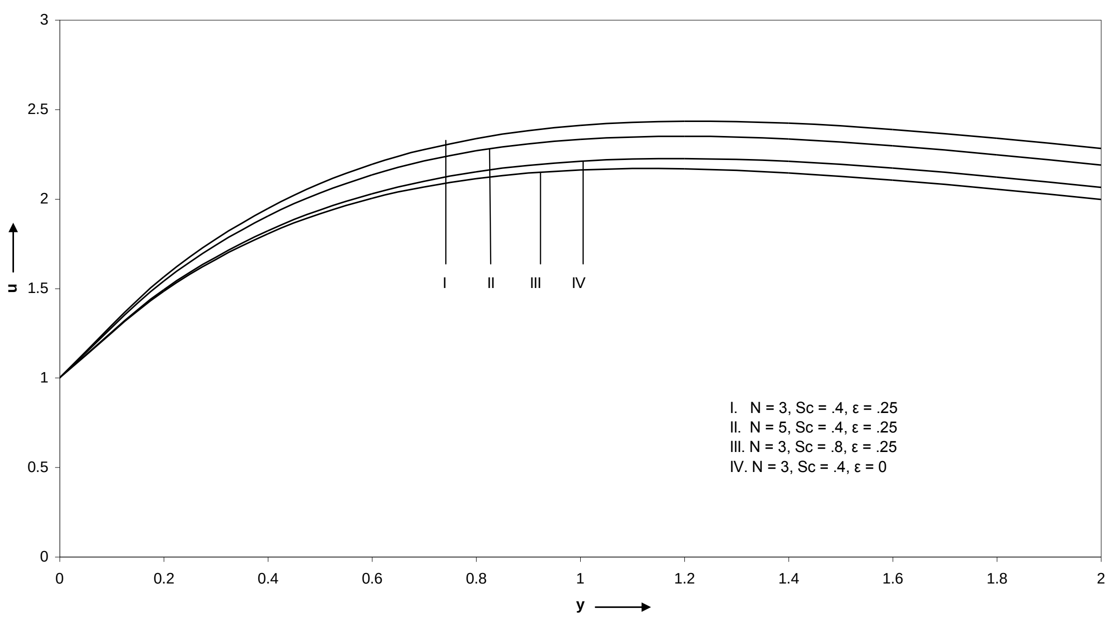
In figure 2, the velocity distribution of boundary layer flow is plotted against y for different ε = .25, M = 2, Gr = 5, Pr = 0.71, Sc = .4, Gc = 3, and ω = 5 and different values of N, K, ωt. It is observed that the fluid velocity increases till y = 1.2, after its velocity decreases continuously with increases in y. It is observed that the fluid velocity decreases due to increasing the radiation parameter N and Phase angle ωt. But fluid velocity increases due to increasing Porosity parameter K.

Fig.2. The velocity distribution for different values of N, K and ωt



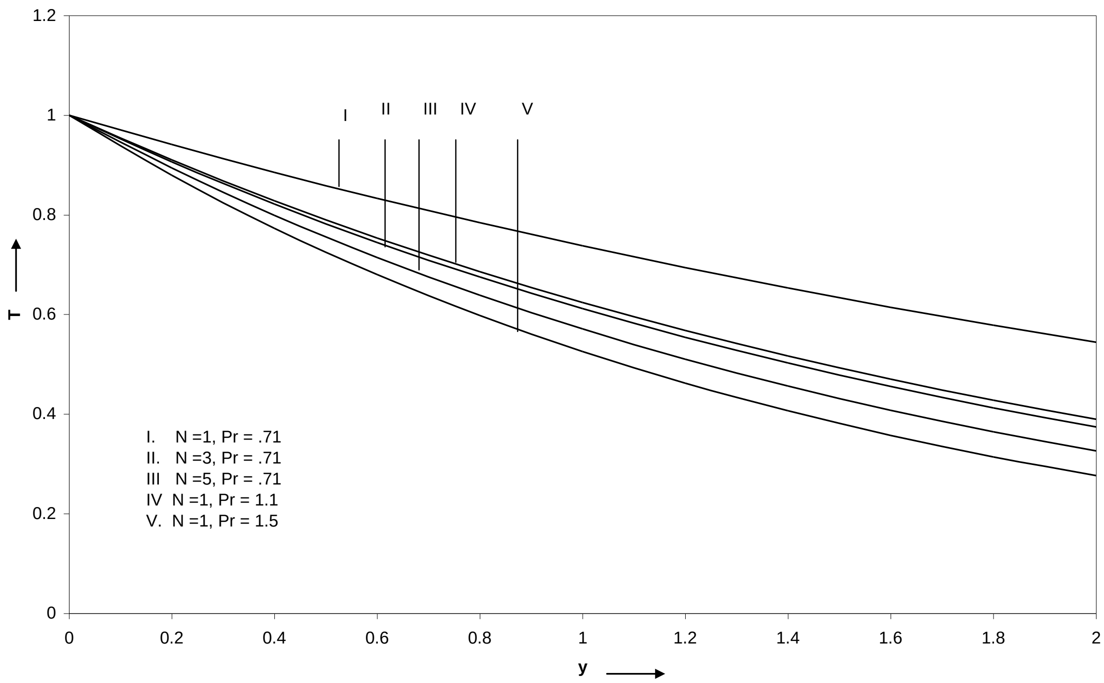
In figure 3, the velocity distribution of boundary layer flow is plotted against y for different K = 3, M = 2, Gr = 5, Pr = 0.71, ωt =π/6, Gc = 3, and ω = 5 and different values of N, K, ωt. It is observed that the fluid velocity increases till y = 1.2, after its velocity decreases continuously with increases in y. It is observed that the fluid velocity decreases due to increasing N, Schmidt number Sc and it is decreases due to decreasing ε.

Fig. 3. The velocity distribution for the different values of N, Sc and ε



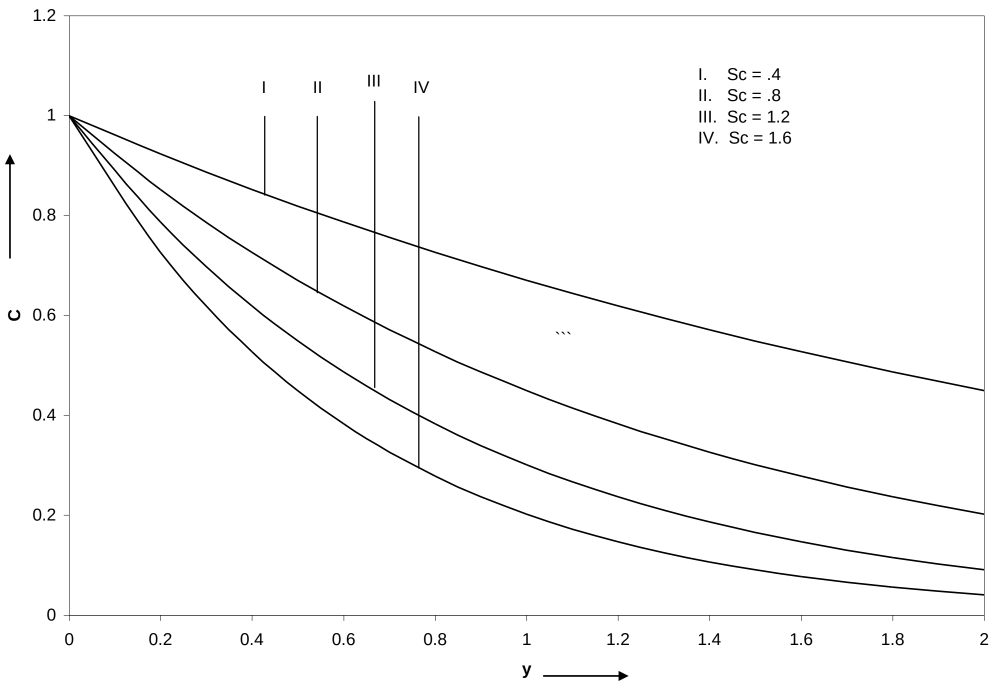
In figure 4, the temperature distribution of boundary layer flow is plotted against y for different ε = .25, M = 2, K = 3 Gr = 5, Sc = .4, Gc = 3, ωt = π/6 and ω = 5 and different values of N and Pr. It is observed that the fluid temperature decreases continuously due to increasing y. it is also observed that the fluid temperature decreases due to increasing the radiation parameter N and Prandtl number Pr.

Fig.4. The temperature distributionfor different value of Pr and N



In figure 5, the concentration distribution of boundary layer flow is plotted against y for different $\varepsilon = .25$, $M = 2$, $K = 3$ $Gr = 5$, $N = 3$, $Gc = 3$, $\omega t = \pi/6$ and $\omega = 5$ and different value of Sc. It is observed that the fluid concentration decreases continuously due to increasing y. it is also observed that the fluid temperature decreases due to increasing the Schmidt number Sc.

Fig.5. The concentration distribution for different values of Sc



In table-1, the skin friction co-efficient and Nusselt number of boundary layer flow are tabulated at y = 0. It is observed that the skin friction co-efficient increases due to increasing in Grashof number Gr, modified Grashof number Gc, and porosity parameter K, frequency parameter ω, and phase angle ωt, while it decreases due to Hartmann number M, and Schmidt number Sc

Table 1: Values of the skin friction co-efficient and Nusselt number at the plate.

Gr	Gc	M	K	N	Sc	ωt	ω	ε	Cf	Nu
5	3	2	3	3	0.4	π/6	5	0	2.964082	0.491538
5	3	2	3	3	0.4	π/6	5	0.25	3.356785	0.491538
10	3	2	3	3	0.4	π/6	5	0.25	5.140058	0.491538
5	5	2	3	3	0.4	π/6	5	0.25	4.14399	0.491538
5	3	3	3	3	0.4	π/6	5	0.25	2.866201	0.491538
5	3	2	5	3	0.4	π/6	5	0.25	3.451002	0.491538
5	3	2	3	5	0.4	π/6	5	0.25	3.225553	0.560526
5	3	2	3	3	0.8	π/6	5	0.25	2.913388	0.491538
5	3	2	3	3	0.4	π/3	5	0.25	3.01274	0.491538
5	3	2	3	3	0.4	π/6	10	0.25	3.376438	0.491538

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