Genetic Algorithm-Based Inventory Model for Deteriorating Items with Inflation and Allowable Payment Delays

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Abstract: The Genetic Algorithm (GA) is a biologically inspired heuristic optimisation method. It has been proven to solve complicated issues effectively. The GA iteratively generates a population of potential solutions, evaluates their fitness, and applies genetic operators including selection, crossover, and mutation to improve the population. The GA has been used in engineering, finance, and bioinformatics to quickly explore huge solution spaces and solve non-linear and non-convex optimisation problems. Thus, the GA is an effective way to locate good candidates. This paper discusses a net commodity with shortages and delays. This commodity cannot set its own price and is always in demand for damaged or unusable goods. This research article reduces system calculation cost using symbol representation, signed distance, and centroid approach. Reduce plotting area costs. This study compares multiple average cost estimates. Numerical examples show the procedure differences. We analyse estimate accuracy and reliability. Our research aims to reveal the best cost estimation approaches. Comprehension tests show these things' cost-effectiveness. This study uses a Genetic Algorithm (GA) to improve network model output and cost estimation.

Keywords: Genetic algorithm, fuzzy triangular number, adaptive representation method, signed distance method and centroid method.

I. INTRODUCTION

To find the minimum value of the inventory problem, which is obtained from the fuzzify model, GA is an algorithmic method. Fuzzification is the process of converting sharp values into rough base values; to achieve these values, several member functions are used. Defuzzification is the process of converting a fuzzy model into a sharp model. According to Zadeh's definition of negative structure from 1965, group work refers to the basic structure and performance such as connection and connection. Park et al. (1987) introduced the notion of fuzzy sets in the EOQ process by utilising a non-zero integer to represent the fast-moving product and streamlining the trading process through the use of the zero-number function based on the extension principle. Kauffman et al developed several functional models in fuzzy logic. in 1991. In their study, Vujosevic et al. (1996) employed fuzzy trapezoidal numbers to partition the cost of the system into the total cost of the commodity-free model, in order to derive a fuzzy total cost. In Yao and Lee's 1996 fuzzy model, the sparse value and fuzzy number are triangular and trapezoidal fuzzy numbers respectively, and the sparse value is a net factor. In Zimmermann's 1996 discussion, net installations and complex objects and their interactions, uses, and functions were discussed. In order to provide further clarification on the matter pertaining to the product, Gen et al. (1997) defined their input data as a set of randomly generated numbers and subsequently formulated an interval mean value model. A random regression method was used by Chang et al. [1998] to estimate non-product problems, which provide triangular fuzzy numbers for non-linear quantities. According to Chang et al. [1999], triangular fuzzy statistics are used in commodity models and fuzzy modeling of commodity prices. The non-commodity inventory model proposed by Lee and Yao (1999) uses triangular fuzzy numbers to approximate the ordered quantities. Yao et al. [2000] constructed a non-deficit commodity model by assuming that linear income and aggregate demand are triangular fuzzy numbers. Understanding the missing individual function and its centroid pattern. Yao and Chiang estimated the expense of goods left out goods in 2003. To crunch the data, they used the centroid and signed distances after converting the total demand and daily holding costs to arbitrary integers. In the presence of non-linear variables, Dutta et al. [2005] develop a model, the optimal solution is generated using a linear input representation method. Chang et al. [2006] extend the model of connecting goods associated with variable delays to delays and lost sales, but at first they make the delay to be a triangular fuzzy number, while their body makes everything that wants to be an irrational triangular number and achieves it. the total number of missing nodes and the input of the centroid method of defuzzification. For below-average products and non-existent products, Wee et al. [2007] presented a real stock model. Using a distance signature method using defuzzification, The inventory issue for a cyclical inventory system with changing lead times and decomposing demand shortages and product costs was established by Lin et al. in their study conducted in 2008. The time is taken as a random integer, and its overall value is determined using a signed optical defuzzification method. In their study Gani and Maheswari (2010) improve the EOQ model using graded average defuzzification. They use fuzzy triangle numbers to depict low-quality items, shortage and defective rates, demand, cost burden, cost projection, and shortage. This method gives researchers an enlarged EOQ model value. A Meli et al. (2011) suggested a signed visual defuzzification inventory approach. This approach addresses the ordering process for low-fault subsurface materials while reducing and optimising cost. Using two defuzzification methods - signed distance and average value - Nezhad et al.(2011) developed a cyclical analysis method in the form of non-stop research analysis, fuzzy processing cost, holding cost, shortage cost, and showed the delay in demand and adding time to the demand of time as variable variables. . A fuzzy model with a conservative limit is described in [2011]. Uthayakumar and Valliathl (2011) developed a business model for Weibull decomposition over infinite space in a dynamic environment. The authors employ triangular fuzzy integers to represent certain cost features and proceed to derive the cost function through the utilisation of the signed distance approach for cost function defuzzification. An inventory model that takes into account demand rates and backlogs is described by Kumar et al. in 2013. The inventory model described by Singh and Sharma [2014] is based on a rough and sharp curve where the scarcity and delay rates are time-dependent. They decomposed the average value and volume measurement using the signed distance method. When the value is worse, Kumar and Rajput [2015] develop a fuzzy index model where the back-end depends on the triangular fuzzy number and the demand depends on the time and the demand rate. . They extracted the total value as a function of time using the signed distance and centroid method. The same fact and those strategies are shown by the research of Kumar and Kumar's [2016] which is not fast, the product depends on the time-reducing factor, and in a different study, they analyze the indicators products for product-based demand. and the right of repatriation. Kumar and Kumar (2016) provide a detailed description of a failed asset inventory model where a GA is used for estimated earnings. In their analysis of the inventory breakdown model of products based on sales volume, Kumar and Agarwal (2016) note that GA is used to reduce the total cost of products while accounting for inflation, shortages and back to back. For an efficient and effective method, Kumar and Kumar [2017] also developed a non-linear commodity model of low-cost and overstated linear demand. In this study, we develop a triangular model of free-order cost, cost containment, damage rate, and cash shortage for the downgrading factor. We used weight representation, signed distance, and centroid defuzzification methods to determine all values; However, the genetic algorithm produces the best results when it is used to maximize the overall cost. With this in mind, the results are categorized as follows using the appropriate quantitative and qualitative analysis. Mondal et al. (2019) created inventory models for degrading and enhancing items in crisp and interval environments. They postulated that the rate of deterioration follows a Weibull distribution with three parameters. In a production inventory model with preservation investment and trade credit policy, Rahman et al. (2020) proposed interval valued deterioration rate. Das et al. (2021) proposed a model for non-instantaneous inventory deterioration with preservation technology and partial backlogged shortages. Paul et al. (2021) examined the effect of default risk of trade credit policy and price-sensitive demand in an inventory model for deteriorating commodities. Paul et al. (2022) analysed the impact of carbon taxation and green investment using an inventory model with green and price-sensitive demand. Mohammad Amin [2023] Environmental and social criteria are incorporated into the formulation of optimal pricing and inventory management decisions for perishable complementary products replenished and sold by the same company.

II. ILLUSTRATION OF FUZZY CONTROL

These details are necessary to deal with the fuzzy commodity model using the fixed distance, the centroid to be adjusted, and the symbol representation.

Illustration 1. (According to Pu and Liu, [25, Illustration 2.1]). If a fuzzy array ã on R=(-∞,∞) possess a membership function of $\mu_a(x)$, it is referred to as a fuzzy point. Aid of the fuzzy array ã is point a in this instance.

$$\mu_a(x) = \begin{Bmatrix} 1, x = a \\ 0, x \neq a \end{Bmatrix} \tag{1}$$

Illustration 2. A fuzzy array [a∝, b∝] 's membership function is $\mu_{[a\propto, b\propto]}(x)$, it is referred to as a level of a fuzzy interval. Where 0≤∝≤ 1 and a<b are established for R.

$$\mu_{[a_\propto, b_\propto]}(x) = \begin{Bmatrix} \alpha, a \le x \le b \\ 0, otherwise \end{Bmatrix}$$

(2)

Illustration 3. A triangular fuzzy number, denoted as $\tilde{A}$ = (a, b, c) is defined as a fuzzy number with a membership function μ_A (x). where a< b < c and established for R.

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{x-b}, b \le x \le c \\ 0, Otherwise \end{cases}$$

(3)

When a=b=c, we have a fuzzy point (c, c, c) = c.

The family of all triangular fuzzy numbers on R is denoted as

$$F_N = \{(a,b,c) \mid a < b < c \forall a,b,c \in R\}$$

Illustration 4.1 If $\tilde{A}$ = (a, b, c) is a triangular fuzzy number and forms the distance $\tilde{A}$ signed as

$$P(\tilde{A}) = \frac{\int_0^{w_A} h\left(\frac{L^{-1}(h) + R^{-1}(h)}{2}\right) dh}{\int_0^{w_A} h dh}$$

With 0< h≤ w_A and 0< w_A ≤ 1

$$P(\tilde{A}) = 1/2 \frac{\int_0^1 h[a+h(b-a)+c-h(c-a)]dh}{\int_0^1 h dh} = \frac{a+4b+c}{6}$$

(4)

Illustration 4.2 If $\tilde{A}$ = (a, b, c) represents a triangular fuzzy number, signed distance of $\tilde{A}$ is established as

$$P(\tilde{A},\tilde{0}) = \int_0^1 d\left[(A_L(\alpha)_\alpha, A_R(\alpha)_\alpha), \tilde{0}\right] = \frac{a+2b+c}{4}$$

(5)

Illustration 4.3 Centroid method applied on triangular fuzzy number $\tilde{A}$ = (a, b, c) is a established as

$$C(\tilde{A}) = \frac{a+b+c}{3}$$

(6)

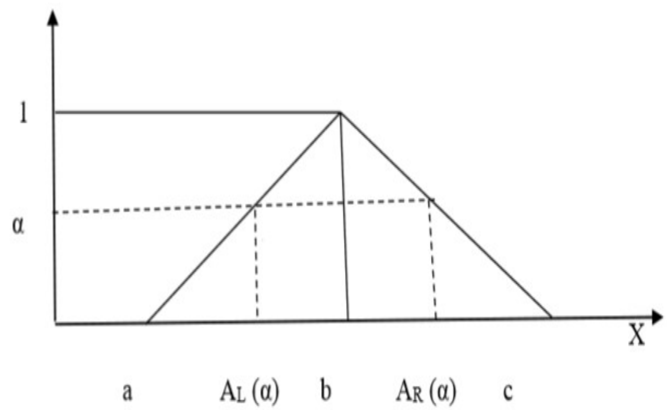

Figure 1: ∝- Carving Representing Triangular Fuzzy Number

III. NOTATIONS AND ASSUMPTIONS

1. Notations

[1] D (t) represents demand rate at any time t per unit time.
[2] A represents ordering cost per order.
[3] ϴ represents decay rate, 0 < ϴ << 1.
[4] T represents length of Cycle.
[5] Q represents ordering quantity per unit.
[6] h is the holding cost per unit per unit time
[7] The deficient rate of time is S.
[8] The rate of time is C.
[9] The overall inventory rate of time is K(t1, T).
[10] The fuzzy required is $\tilde{D}$.

[11] Fuzzy decaying rate $\tilde{\theta}$.

[12] Fuzzy purchasing expense per unit of time is $\tilde{h}$.

[13] Fuzzy shortage rate of time is $\tilde{a}$.

[14] Fuzzy unit rate of time is $\tilde{S}$.

[15] $\tilde{C}$ denotes total fuzzy inventory rate of time.

[16] $K_{dG}(t_1,T)$ denotes defuzzify value of $\tilde{K}_{dG}(t_1,T)$ using the Graded mean integration method.

[17] $K_{dS}(t_1,T)$ denotes defuzzify value of $\tilde{K}_{dS}(t_1,T)$ using the Signed-distance method

[18] K_{dC} (t_1, T) denotes defuzzify value of $\tilde{K}_{dC}(t_1,T)$ using the centroid method.

2. Assumptions

(i) There is a limit to the length of time.
(ii) Lead time is zero and replenishment occurs instantly.

(iii) Backlogs and shortages are both permitted. Demand that hasn't been met yet is backordered, and the percentage of shortages that are backordered is $1/(1+\delta(T-t))$, here δ is a positive constant.

(iv) The Constant demand rate $D(t) = a$ is considered. Where a is positive Constants and $a \geq 0$.

(v) There is no occurrence of material repair during the cycle.

IV. INVENTORY MODEL LAYOUT

It's assumed q(t) represents the inventory that is available at time t having initial stock Q. The inventory level gradually drops between [0, t1] due to customer satisfaction, market demand, and product degradation, and the subsequent period [t1, T] is characterized by partially backlogged shortages. The differential equations determine the inventory level q(t) at any instant in time.

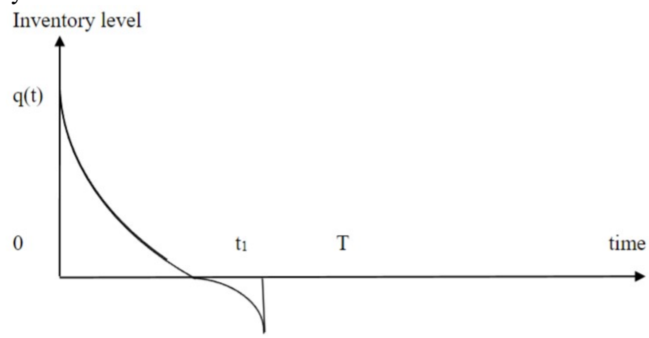

Figure 2: Graphical Representation of Inventory Model

V. Crisp Model and Fuzzy Model

Crisp Model

Assume that q(t) represents the inventory level at any given time, which is controlled by the following two differential equations:

$$\frac{d\,q(t)}{dt} + \theta\, q(t) = -a \qquad 0 \leq t \leq t_1 \qquad (7)$$

With q(0) = Q and $q(t_1) = 0$.

$$\frac{d\,q(t)}{dt} = -\frac{(a)}{1+\delta(T-t)} \qquad t_1 \leq t \leq T \qquad (8)$$

With $q(t_1) = 0$.

The solution of equations (7) and (8) is given by

$$q(t) = (1-\theta t)\left[Q - a\left(t + \frac{t^2}{2}\right)\right] \qquad (9)$$

$$q(t) = \left(\frac{a}{\delta}\right)\left[\log\{1+\delta(T-t)\} - \log\{1+\delta(T-t_1)\}\right] \qquad (10)$$

By putting $q(t_1) = 0$, we get

$$Q = a\left(t_1 + \frac{t_1^{\,2}}{2}\right)$$

(11)

Now, formula (10) develops into

$$q(\mathrm{t}) = \left[a\left\{\left(t_1 + \frac{t_1^{\,2}}{2}\right) - \left(t + \frac{t^2}{2}\right)\right\} - a\theta t\left\{\left(t_1 + \frac{t_1^{\,2}}{2}\right) - \left(t + \frac{t^2}{2}\right)\right\}\right]$$

(12)

(Ignoring higher powers of θ).

Total average no. of holding units (I_H) in the interval [0, T] is presented using

$$I_H = \int_0^{t_1} \mathrm{q(t)\,dt} = \left[a\left(\frac{t_1^{\,2}}{2} + \frac{t_1^{\,3}}{3}\right) - a\theta\left(\frac{t_1^{\,4}}{8} + \frac{t_1^{\,3}}{6}\right)\right]$$

(13)

Total no. of decayed units (I_D) in the interval [0, T] is presented using

$$I_D = Q - Total\ Demand$$

$$I_D = Q - \int_0^{t_1} a\,\mathrm{dt} = \frac{at_1^{\,2}}{2}$$

(14)

Total average no. of shortage units (I_S) in the interval [0, T] is presented using

$$I_s = -\int_{t_1}^{T} \frac{a}{1+\delta\left(T-t\right)}\mathrm{dt} = \left(\frac{a}{\delta}\right)\left[\log T - (T-t_1) + \frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \quad (15)$$

Total cost of the system per unit time is presented using

$$K\left(t_1,\ T\right) = \frac{1}{T}\left[A + h\,I_H + C\,I_D + S\,I_S\right]$$

$$K\left(t_1,\ T\right) = \frac{1}{T}\left[A + h\left\{a\left(\frac{t_1^{\,2}}{2} + \frac{t_1^{\,3}}{3}\right) - a\theta\left(\frac{t_1^{\,4}}{8} + \frac{t_1^{\,3}}{6}\right)\right\}\right.$$

$$\left. + C\left(\frac{at_1^{\,2}}{2}\right) + S\left\{\left(\frac{a}{\delta}\right)\left[\log T - (T-t_1) + \frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\right\}\right]$$

(16)

Fuzzy Model

The rate is always considered in the development of eq forms by previous authors. In Crisp during the Crisp developed, it was thought to be decided or pointed out apparently; However, in reality conditions, due to unarmed environment, it is difficult to explain all parameters. Therefore, we think that some variables, ie $\tilde{a}$, $\tilde{C}$, $\tilde{S}$, $\tilde{\theta}$, $\tilde{h}$, can develop under certain limits.

Let $\tilde{a} = (a_1, a_2, a_3), \tilde{C} = (C_1, C_2, C_3), \tilde{S} = (S_1, S_2, S_3),\ \tilde{\theta} = (\theta_1, \theta_2, \theta_3),\ \tilde{h} = (h_1, h_2, h_3),$ are as triangular fuzzy numbers.

The system's overall rate of time, expressed in a fuzzy style, is provided by

$$K(t_1, T) = \frac{1}{T}\left[\begin{array}{l} A + \tilde{h}\tilde{a}\left(\frac{t_1^2}{2} + \frac{t_1^3}{3}\right) - \tilde{h}\tilde{a}\tilde{\theta}\left(\frac{t_1^4}{8} + \frac{t_1^3}{6}\right) \\ + \tilde{C}\left(\frac{\tilde{a}t_1^2}{2}\right) + \tilde{S}\left(\frac{\tilde{a}}{\delta}\right)\left[\log T - (T - t_1) + \frac{1}{\delta}\log\left\{1 + \delta\left(T - t_1\right)\right\}\right] \end{array}\right] \tag{17}$$

The graded mean representation, signed distance, and centroid method's fuzzy total cost is defuzzified.

(1) Total Cost is provided via the **Graded Mean Representation** Method as follows:

$$K_{dG}\left(t_1, T\right) = \frac{1}{6}\left[K_{dG_1}\left(t_1, T\right), K_{dG_2}\left(t_1, T\right), K_{dG_3}\left(t_1, T\right)\right]$$

Where

$$K_{dG_1}(t_1, T) = \frac{1}{T}\left[\begin{array}{l} A + h_1 a_1\left(\frac{t_1^2}{2} + \frac{t_1^3}{3}\right) - h_1 a_1 \theta_1\left(\frac{t_1^4}{8} + \frac{t_1^3}{6}\right) \\ + C_1\left(\frac{a_1 t_1^2}{2}\right) + S_1\left(\frac{a}{\delta}\right)\left[\log T - (T - t_1) + \frac{1}{\delta}\log\left\{1 + \delta\left(T - t_1\right)\right\}\right] \end{array}\right] \tag{18}$$

$$K_{dG_2}(t_1, T) = \frac{1}{T}\left[\begin{array}{l} A + h_2 a_2\left(\frac{t_1^2}{2} + \frac{t_1^3}{3}\right) - h_2 a_2 \theta_2\left(\frac{t_1^4}{8} + \frac{t_1^3}{6}\right) \\ + C_2\left(\frac{a_2 t_1^2}{2}\right) + S_2\left(\frac{a}{\delta}\right)\left[\log T - (T - t_1) + \frac{1}{\delta}\log\left\{1 + \delta\left(T - t_1\right)\right\}\right] \end{array}\right] \tag{19}$$

$$K_{dG_3}(t_1, T) = \frac{1}{T}\left[\begin{array}{l} A + h_3 a_3\left(\frac{t_1^2}{2} + \frac{t_1^3}{3}\right) - h_3 a_3 \theta_3\left(\frac{t_1^4}{8} + \frac{t_1^3}{6}\right) \\ + C_3\left(\frac{a_3 t_1^2}{2}\right) + S_3\left(\frac{a}{\delta}\right)\left[\log T - (T - t_1) + \frac{1}{\delta}\log\left\{1 + \delta\left(T - t_1\right)\right\}\right] \end{array}\right] \tag{20}$$

$$K_{dG}\left(t_1, T\right) = \frac{1}{6}\left[K_{dG_1}\left(t_1, T\right) + 4K_{dG_2}\left(t_1, T\right) + K_{dG_3}\left(t_1, T\right)\right] \tag{21}$$

So

$$K_{dG}\left(t_1,T\right)=\frac{1}{6T}\left[\begin{array}{l} A+h_1a_1\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)-h_1a_1\theta_1\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right) \\ +C_1\left(\frac{a_1t_1^2}{2}\right)+S_1\left(\frac{a}{\delta}\right)\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]+ \\ 4\left\{\begin{array}{l} A+h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)-h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right) \\ +C_2\left(\frac{a_2t_1^2}{2}\right)+S_2\left(\frac{a}{\delta}\right)\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \end{array}\right\}+ \\ A+h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)-h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right) \\ +C_3\left(\frac{a_3t_1^2}{2}\right)+S_3\left(\frac{a}{\delta}\right)\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \end{array}\right]$$

(22)

By resolving the following equations, the ideal value of t1 and T can be determined in order to minimize total cost function per unit time $K_{dG_1}\left(t_1,T\right)$:

$$\frac{\partial K_{dG}\left(t_1,T\right)}{\partial t_1}=0$$

(23)

$$\frac{\partial K_{dG}\left(t_1,T\right)}{\partial T}=0;$$

Equation (23) is equivalent to.

$$\frac{1}{6T}\left[\begin{array}{l} h_1a_1\left(t_1+t_1^2\right)-h_1a_1\theta_1\left(\frac{t_1^3}{2}+\frac{t_1^2}{2}\right)+C_1a_1t_1+4\left\{A+h_2a_2\left(t_1+t_1^2\right)-h_2a_2\theta_2\left(\frac{t_1^3}{2}+\frac{t_1^2}{2}\right)+C_2a_2t_1\right\}+ \\ h_3a_3\left(t_1+t_1^2\right)-h_3a_3\theta_3\left(\frac{t_1^3}{2}+\frac{t_1^2}{2}\right)+C_3a_3t_1 \end{array}\right]=0$$

(24)

$$\left[\begin{array}{l}\frac{1}{6T}\left[\left\{\frac{S_1a_1}{\delta T}+\frac{S_1a_1}{\delta}-\frac{S_3a_3}{\delta}\right\}+\left\{\frac{S_2a_2}{\delta T}+\frac{S_2a_2}{\delta}-\frac{S_2a_2}{\delta}\right\}+\left\{\frac{S_3a_3}{\delta T}+\frac{S_3a_3}{\delta}-\frac{S_1a_1}{\delta}\right\}\right] \\ -\frac{1}{6T^2}\left\{\begin{array}{l}\left[6A+h_1a_1\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_1a_1\theta_1\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_1\frac{a_1t_1^2}{2}\right] \\ S_1\frac{a_1}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]+ \\ 4\left[\begin{array}{l}h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_2\frac{a_2t_1^2}{2}+ \\ S_2\frac{a_2}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]+ \\ h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_3\frac{a_3t_1^2}{2}+ \\ S_3\frac{a_3}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right\}\end{array}\right]=0 \tag{25}$$

Further, for the overall price function $K_{dG}\left(t_1,T\right)$, to be convex, the mentioned constraints must be fullfiled

$$, \quad \frac{\partial^2 K_{dG}\left(t_1,T\right)}{\partial T^2}>0; \quad \frac{\partial^2 K_{dG}\left(t_1,T\right)}{\partial t_1{}^2}>0 \tag{26}$$

And $$\left(\frac{\partial^2 K_{dG}\left(t_1,T\right)}{\partial t_1{}^2}\right)\left(\frac{\partial^2 K_{dG}\left(t_1,T\right)}{\partial T^2}\right)-\left(\frac{\partial^2 K_{dG}\left(t_1,T\right)}{\partial t_1\partial T}\right)>0; \tag{27}$$

The overall price function $K_{dG}\left(t_1,T\right)$'s second derivatives are intricate, making it challenging t mathematically demonstrate the convexity.

(ii) Using **Signed Distance Method**, Overall price is presented using

$$K_{dS}\left(t_1,T\right)=\frac{1}{4}\left[K_{dS_1}\left(t_1,T\right),K_{dS_2}\left(t_1,T\right),K_{dS_3}\left(t_1,T\right)\right]$$

Where

$$K_{dS_1}\left(t_1,\ T\right)=\frac{1}{T}\left[\begin{array}{l}A+h_1a_1\left(\frac{t_1{}^2}{2}+\frac{t_1{}^3}{3}\right)+h_1a_1\theta_1\left(\frac{t_1{}^4}{8}+\frac{t_1{}^3}{6}\right)+C_1\left(\frac{a_1t_1{}^2}{2}\right) \\ +S_1\left(\frac{a}{\delta}\right)\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right] \tag{28}$$

$$K_{dS_2}\left(t_1,\ T\right)=\frac{1}{T}\left[\begin{array}{l}A+h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_2\left(\frac{a_2t_1^2}{2}\right)\\+S_2\frac{a_2}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right] \tag{29}$$

$$K_{dS_3}\left(t_1,\ T\right)=\frac{1}{T}\left[\begin{array}{l}A+h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_3\left(\frac{a_3t_1^2}{2}\right)\\S_3\frac{a_3}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right] \tag{30}$$

$$K_{dS}\left(t_1,T\right)=\frac{1}{4}\left[K_{dS_1}\left(t_1,T\right)+2K_{dS_2}\left(t_1,T\right)+K_{dS_3}\left(t_1,T\right)\right] \tag{31}$$

So

$$K_{dS}\left(t_1,T\right)=\frac{1}{4T}\left[\begin{array}{l}\left[\begin{array}{l}A+h_1a_1\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_1a_1\theta_1\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_1\left(\frac{a_1t_1^2}{2}\right)\\+S_1\left(\frac{a}{\delta}\right)\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]+\\2\left[\begin{array}{l}A+h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_2\left(\frac{a_2t_1^2}{2}\right)\\+S_2\frac{a_2}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]+\\\left[\begin{array}{l}A+h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_3\left(\frac{a_3t_1^2}{2}\right)\\S_3\frac{a_3}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]\end{array}\right] \tag{32}$$

Utilising the same methodology as in example (i), the overall price function $K_{dS}\left(t_1,T\right)$ has been minimized. The ideal estimate of t1 and T for reducing the overall price function per unit time $K_{dS}\left(t_1,T\right)$ can be found by evaluating the equations mention below:

$$\frac{\partial K_{dS}\left(t_1,T\right)}{\partial t_1}=0 \quad and \quad \frac{\partial K_{dS}\left(t_1,T\right)}{\partial T}=0;$$

(33)

Equation (32) is equivalent to

$$\frac{1}{4T}\left[\begin{array}{l} h_1a_1\left(t_1+t_1^2\right)+h_1a_1\theta_1\left(\frac{t_1^3}{2}+\frac{t_1^2}{2}\right)+C_1a_1t_1+ \\ 2\left\{h_2a_2\left(t_1+t_1^2\right)+h_2a_2\theta_2\left(\frac{t_1^3}{2}+\frac{t_1^2}{2}\right)+C_2a_2t_1\right\}+ \\ h_3a_3\left(t_1+t_1^2\right)+h_3a_3\theta_3\left(\frac{t_1^3}{2}+\frac{t_1^2}{2}\right)+C_3a_3t_1 \end{array}\right]=0 \quad (34)$$

And

$$\frac{1}{4T}\left\{\begin{array}{l} \left[\frac{S_1a_1}{\delta T}+\frac{S_1a_1}{\delta}-\frac{S_3a_3}{\delta}\right]+2\left[\frac{S_2a_2}{\delta T}+\frac{S_2a_2}{\delta}-\frac{S_2a_2}{\delta}\right]+\left[\frac{S_3a_3}{\delta T}+\frac{S_3a_3}{\delta}-\frac{S_1a_1}{\delta}\right] \\ -\frac{1}{4T^2}\left[\begin{array}{l} 6A+h_1a_1\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_1a_1\theta_1\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_1\frac{a_1t_1^2}{2} \\ +S_1\frac{a}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \\ +2\left[h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)\right]+C_2\frac{a_2t_1^2}{2}+ \\ S_2\frac{a_2}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \\ h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_3\frac{a_3t_1^2}{2}+ \\ S_3\frac{a_3}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \end{array}\right] \end{array}\right\}=0 \quad (35)$$

Further, for the overall price function $K_{dG}\left(t_1,T\right)$, to be convex, the mentioned constraints must be fulfilled

$$\frac{\partial^2 K_{dS}\left(t_1,T\right)}{\partial t_1^{\,2}}>0 \;, \quad \frac{\partial^2 K_{dS}\left(t_1,T\right)}{\partial T^2}>0; \quad (36)$$

And $\left(\frac{\partial^2 K_{dS}\left(t_1,T\right)}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 K_{dS}\left(t_1,T\right)}{\partial T^2}\right)-\left(\frac{\partial^2 K_{dS}\left(t_1,T\right)}{\partial t_1\partial T}\right)>0;$

(37)

The overall price function $K_{dS}\left(t_1,T\right)$'s second derivatives are intricate, making it challenging t mathematically demonstrate the convexity.

(iii) Using **centroid Method**, the overall price is presented using

$$K_{dC}\left(t_1,T\right)=\frac{1}{3}\left[K_{dC_1}\left(t_1,T\right),K_{dC_2}\left(t_1,T\right),K_{dC_3}\left(t_1,T\right)\right]$$

Where

$$K_{dC_1}\left(t_1,\ T\right)=\frac{1}{T}\left[\begin{array}{l}A+h_1a_1\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_1a_1\theta_1\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_1\frac{a_1t_1^2}{2}\\+S_1\frac{a}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right] \quad (38)$$

$$K_{dC_2}\left(t_1,\ T\right)=\frac{1}{T}\left[\begin{array}{l}A+h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_2\frac{a_2t_1^2}{2}\\+S_2\frac{a_2}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right] \quad (39)$$

$$K_{dC_3}\left(t_1,\ T\right)=\frac{1}{T}\left[\begin{array}{l}A+h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_3\frac{a_3t_1^2}{2}\\+S_3\frac{a_3}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right] \quad (40)$$

$$K_{dC}\left(t_1,T\right)=\frac{1}{3}\left[K_{dC_1}\left(t_1,T\right)+K_{dC_2}\left(t_1,T\right)+K_{dC_3}\left(t_1,T\right)\right] \quad (41)$$

So

$$K_{dC}\left(t_1,\,T\right)=\frac{1}{3T}\left[\begin{array}{l}\left[\begin{array}{l}A+h_1a_1\left(\dfrac{t_1^{\,2}}{2}+\dfrac{t_1^{\,3}}{3}\right)+h_1a_1\theta_1\left(\dfrac{t_1^{\,4}}{8}+\dfrac{t_1^{\,3}}{6}\right)+C_1\dfrac{a_1t_1^{\,2}}{2}\\+S_1\dfrac{a}{\delta}\left[\log T-(T-t_1)+\dfrac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]+\\\left[\begin{array}{l}A+h_2a_2\left(\dfrac{t_1^{\,2}}{2}+\dfrac{t_1^{\,3}}{3}\right)+h_2a_2\theta_2\left(\dfrac{t_1^{\,4}}{8}+\dfrac{t_1^{\,3}}{6}\right)+C_2\dfrac{a_2t_1^{\,2}}{2}\\+S_2\dfrac{a_2}{\delta}\left[\log T-(T-t_1)+\dfrac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]+\\\left[\begin{array}{l}A+h_3a_3\left(\dfrac{t_1^{\,2}}{2}+\dfrac{t_1^{\,3}}{3}\right)+h_3a_3\theta_3\left(\dfrac{t_1^{\,4}}{8}+\dfrac{t_1^{\,3}}{6}\right)+C_3\dfrac{a_3t_1^{\,2}}{2}\\+S_3\dfrac{a_3}{\delta}\left[\log T-(T-t_1)+\dfrac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right]\end{array}\right]\end{array}\right] \tag{42}$$

Utilising the same methodology as in example (i), the overall price function $K_{dC}\left(t_1,T\right)$ has been minimized. The ideal estimate of t1 and T for reducing the overall price function per unit time $K_{dC}\left(t_1,T\right)$ can be found by evaluating the equations mention below:

$$\frac{\partial K_{dC}\left(t_1,T\right)}{\partial t_1}=0 \quad and \quad \frac{\partial K_{dC}\left(t_1,T\right)}{\partial T}=0; \tag{43}$$

Equation (42) is similar to

$$\frac{1}{3T}\left[\begin{array}{l}h_1a_1\left(t_1+t_1^{\,2}\right)+h_1a_1\theta_1\left(\dfrac{t_1^{\,3}}{2}+\dfrac{t_1^{\,2}}{2}\right)+C_1a_1t_1+\\h_2a_2\left(t_1+t_1^{\,2}\right)+h_2a_2\theta_2\left(\dfrac{t_1^{\,3}}{2}+\dfrac{t_1^{\,2}}{2}\right)+C_2a_2t_1+\\h_3a_3\left(t_1+t_1^{\,2}\right)+h_3a_3\theta_3\left(\dfrac{t_1^{\,3}}{2}+\dfrac{t_1^{\,2}}{2}\right)+C_3a_3t_1\end{array}\right]=0 \tag{44}$$

And

$$\left[\begin{array}{l} \frac{1}{3T}\left\{\left(\frac{S_1a_1}{\delta T}+\frac{S_1a_1}{\delta}-\frac{S_3a_3}{\delta}\right)+\left(\frac{S_2a_2}{\delta T}+\frac{S_2a_2}{\delta}-\frac{S_2a_2}{\delta}\right)+\left(\frac{S_3a_3}{\delta T}+\frac{S_3a_3}{\delta}-\frac{S_1a_1}{\delta}\right)\right\} \\ -\frac{1}{3T^2}\left\{\begin{array}{l} \left(\begin{array}{l} 3A+h_1a_1\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_1a_1\theta_1\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_1\frac{a_1t_1^2}{2} \\ +S_1\frac{a}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \end{array}\right) \\ +\left(\begin{array}{l} h_2a_2\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_2a_2\theta_2\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_2\frac{a_2t_1^2}{2} \\ +S_2\frac{a_2}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \end{array}\right) \\ +\left(\begin{array}{l} h_3a_3\left(\frac{t_1^2}{2}+\frac{t_1^3}{3}\right)+h_3a_3\theta_3\left(\frac{t_1^4}{8}+\frac{t_1^3}{6}\right)+C_3\frac{a_3t_1^2}{2}+ \\ S_3\frac{a_3}{\delta}\left[\log T-(T-t_1)+\frac{1}{\delta}\log\left\{1+\delta\left(T-t_1\right)\right\}\right] \end{array}\right) \end{array}\right\} \end{array}\right]=0 \tag{45}$$

Further, for the overall price function $K_{dC}\left(t_1,T\right)$, to be convex, the mentioned constraints must be fullfiled

$$\frac{\partial^2 K_{dC}\left(t_1,T\right)}{\partial t_1^{\,2}}>0 \quad , \quad \frac{\partial^2 K_{dC}\left(t_1,T\right)}{\partial T^2}>0; \tag{46}$$

And $$\left(\frac{\partial^2 K_{dC}\left(t_1,T\right)}{\partial t_1^{\,2}}\right)\left(\frac{\partial^2 K_{dC}\left(t_1,T\right)}{\partial T^2}\right)-\left(\frac{\partial^2 K_{dC}\left(t_1,T\right)}{\partial t_1\partial T}\right)>0; \tag{47}$$

The overall price function $K_{dC}\left(t_1,T\right)$'s second derivatives are intricate, making it challenging t mathematically demonstrate the convexity.

VI. NUMERICAL EXAMPLES

Let us consider an inventory system that is characterised by the following parametric values.

Crisp Model

A= Rs180 /order, C = Rs18 /unit, h =Rs. 4/unit/year, a =100 units/year, θ =0.01 /year, S =Rs 13 /unit/year.

The solution of crisp model is $K\left(t_1,T\right)$ = Rs 393.2924, t_1 = 0.6447 year, T = 0.8695 year.

Fuzzy Model

When $\tilde{a},\tilde{C},\tilde{S},\tilde{\theta},\tilde{h}$ all are triangular fuzzy numbers where

$\tilde{a}$ = (48, 80, 112), $\tilde{C}$ = (15, 20, 22), $\tilde{S}$ = (12, 14, 16), $\tilde{\theta}$ = (0.006, 0.010, 0.012), $\tilde{h}$ = (3, 4, 6)

Fuzzy model can be solved using below mentioned three ways:

Table 1. The Fuzzy Model is determined by the utilisation of three distinct methodologies.

S. No.	Graded Mean Representation Method			Signed Distance Method			centroid Method		
	t_1 (In Year)	T (In Year)	$K_{dG}(t_1, T)$ (In Rs.)	t_1 (In Year)	T (In Year)	$K_{dS}(t_1, T)$ (In Rs.)	t_1 (In Year)	T (In Year)	$K_{dC}(t_1, T)$ (In Rs.)
1.	0.5407	0.9028	392.1084	0.6408	0.8395	412.5096	0.6265	0.8887	417.6586
2.	0.7135	0.9368	384.2552	0.6735	0.8362	407.7852	0.7055	0.9263	409.9852
3.	0.6915	0.8394	383.5274	0.6915	0.8496	398.6274	0.6715	0.8492	405.8284
4.	0.6095	0.8513	380.6578	0.6110	0.8508	397.5264	0.6718	0.8503	404.2691

Vulnerability Inquiry

By using the defuzzified values of these parameters, a Vulnerability Inquiry is being executed to investigate the implications of modifications in the fuzzy variables $\tilde{\alpha}$ and $\tilde{\theta}$ on the ideal solution. The tables below display the findings.

Table 2. Vulnerability Investigation of Variable "a"

a (units/year)	t_1 (year)	T (year)	$K_{dG}(t_1, T)$ (Rs)
50	0.841	1.164	316.646
70	0.732	1.024	357.148
90	0.640	0.918	416.749
110	0.614	0.851	455.747
130	0.549	0.764	489.359

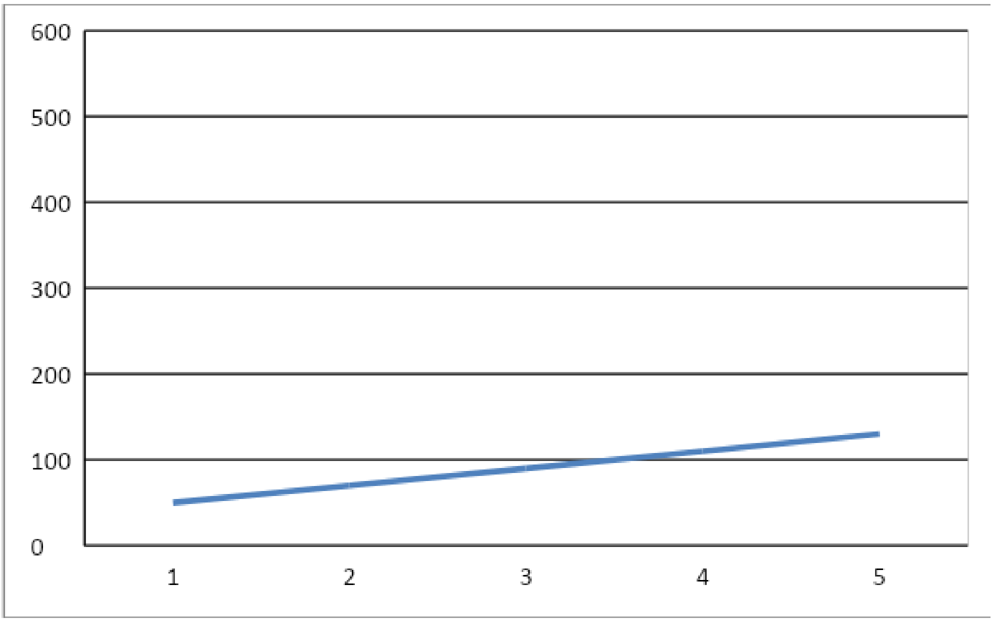

Figure 3. The influence of demand parameter "a" on the total fuzzy cost.

Table 3. Vulnerability Investigation of Variable θ

θ	t_1 **(year)**	**T (year)**	$K_{dG}(t_1, T)$ **(Rs)**
0.005	0.687	0.934	411.248
0.007	0.684	0.934	412.416
0.009	0.670	0.918	414.627
0.011	0.664	0.915	415.686
0.013	0.669	0.912	416.968

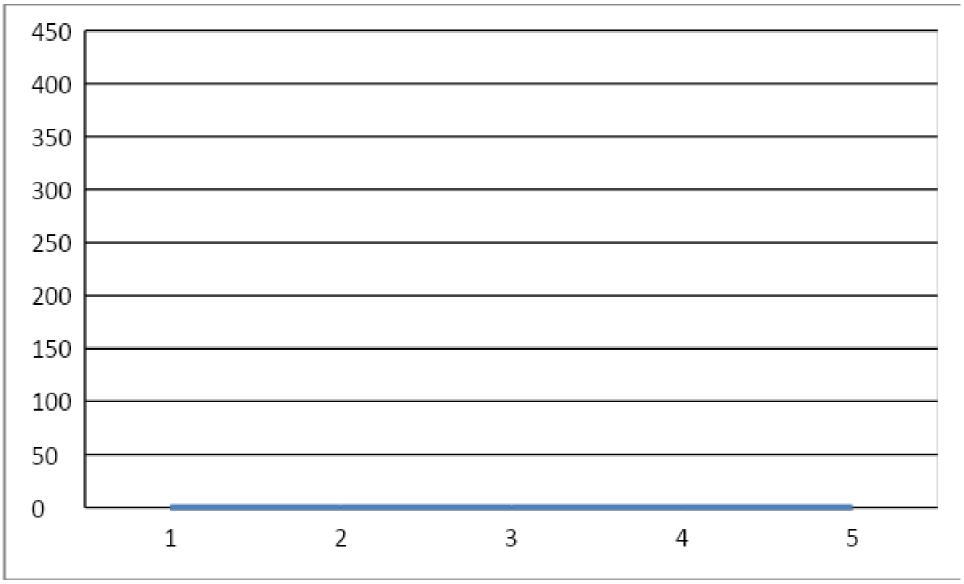

Figure 4. The effect of demand parameter θ on total fuzzy cost.

Observations

1. If the values of a and are simultaneously increased while the ideal rates of t1 and T are decreased, the overall fuzzy value $K_{dG}(t_1, T)$ exhibits an increase. We can clearly see from the Table 2 and Figure 3.
2. If the values of θ and are simultaneously increased while the optimum rates of t1 and T are decreased, the overall fuzzy value $K_{dG}(t_1, T)$ will increase. We can clearly see from the Table 3 and Figure 4.

VII. IMPLEMENTATION OF GENETIC ALGORITHM

The basis for Genetic Algorithm (GA) is "Darwin's Theory". This is a theory for discovering an optimized solution from a set of solutions, not a method for solving a problem. We can use this algorithm as long as we're not happy with the results or until we obtain an optimized result.
The GA finds an optimized solution using three main techniques.

The three main methods are:

1. Selection: In this method we select a new generation using fitness function from existing population and called as 'Parent'. We need parent to find next effective generation.
2. Crossover: The subsequent approach involves the utilisation of crossover, wherein crossover operations are performed to generate a new generation (referred to as "Children") from the existing generation (comprising of two Parents) obtained through the selection process.

Mutation: Mutation, the last method, randomly alters parents picked via selection and crossover. Find a valuable new breed or generation.

We are applying Genetic algorithm in Table 2: Vulnerability Inquiry on variables a.

Table 4. Solution after Applying GA in Vulnerability Inquiryon variables a

a **(units/year)**	t_1 **(year)**	**T (year)**	$K_{dG}(t_1, T)$ **(Rs)**
50	0.7326	1.0249	315.6532
70	0.8412	1.1642	347.1471

90	0.6146	0.8514	402.7569
110	0.6402	0.9182	448.8772
130	0.5498	0.7646	474.3754

And again we are applying GA in Table 3. Vulnerability Inquiry on variables θ

Table 5. Solution after applying GA in Vulnerability Inquiry on variables θ

θ	t_1 **(year)**	**T (year)**	$K_{dG}(t_1, T)$ **(Rs)**
0.005	0.6846	0.9347	409.2452
0.007	0.6873	0.9342	411.2583
0.009	0.6644	0.9154	412.2457
0.011	0.6705	0.9184	413.4567
0.013	0.6696	0.9121	415.5782

We can see the effective changes in overall price (OP) after applying crossover and mutation methods.

VII. CONCLUSION

This study uses triangular fuzzy numbers to represent deterioration, holding costs, scarcity costs, and demand in its inventory model. T, the cycle's duration, equals t1, the ideal stock time period. Three methods—centroid, signed distance, and graded mean representation—defuzzify. We analyse varying values of a using numerical examples and a Vulnerability Inquiry before utilising a genetic algorithm to minimise cost.

We wrote code in the Java programming language to find values similar to T (Length of the cycle), t (time), $K_{dG}(t_1,T)$, $K_{dS}(t_1,T)$, K_{dC} (t_1, T), etc. In order to analyse the Genetic Algorithm, Matlab 2016b is employed. The aforementioned framework can be developed to include time- and stock-dependent demand, among other things.

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